Scattergraph Principles and Practice

Camp's Varying Roughness Coefficient Applied to the Manning Equation

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ABSTRACT

The Manning Equation is an empirical formula commonly used to design sewer systems. Most design methods assume that the roughness coefficient is constant, but historical research has shown that it varies as a function of flow depth. The use of a constant or varying roughness coefficient is often left to the discretion of the design engineer.

The same consideration applies to scattergraph methods that correlate the Manning Equation to flow monitor data. The Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method have been previously reported using a constant roughness coefficient. A modification is presented in this paper to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%.

Examples of these methods using a constant and varying roughness coefficient are provided from flow monitor locations throughout the United States. Laboratory research by the authors is also provided and indicates that the use of a varying roughness coefficient provides a more accurate determination of sewer capacity.

KEY WORDS Flow Monitoring, Manning Equation, Scattergraph, Roughness Coefficient

Introduction

The scattergraph is a graphical tool that provides insight into sewer performance through a simple and intuitive display of flow monitor data. The resulting patterns form characteristic signatures that reveal important information about conditions within a sewer and the impact that these conditions have on sewer capacity.[1] The Manning Equation is an important component of the scattergraph and can be applied using a variety of methods. The Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method have been previously reported using a constant roughness coefficient.[2] A modification is presented to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%.

Manning Equation

The Manning Equation is an empirical formula used to design sewer systems. The most common expression of this formula is provided in Equation (1).

$$v = \frac{1}{n} R^{2/3} S^{1/2} \qquad (1)$$

where: v = flow velocity, m/s
n = roughness coefficient
R = hydraulic radius, m
S = slope of the energy gradient

Several assumptions are generally made with respect to the Manning Equation: the roughness coefficient is constant, and the slope of the energy gradient equals the slope of the pipe.[3] However, historical research reported by Camp and others has shown that the roughness coefficient varies as a function of flow depth.[4] This variation can be expressed in general terms as provided in Equation (2).

$$n = n_D f(d) \qquad (2)$$

where: n = roughness coefficient
n_D = roughness coefficient at d = D

The varying roughness coefficient is incorporated into the Manning Equation by direct substitution as shown in Equation (3).

$$v = \frac{1}{n_D f(d)} R^{2/3} S^{1/2} \qquad (3)$$

where: v = flow velocity, m/s
n_D = roughness coefficient at d = D
R = hydraulic radius, m
S = slope of the energy gradient

Based on this revised assumption, the Manning Equation can be algebraically rearranged such that the constant parameters are consolidated into a single coefficient, defined as the *hydraulic coefficient*, and restated as provided in Equation (4). This expression is useful in subsequent discussions.

$$v = \frac{C}{f(d)} R^{2/3} \qquad (4)$$

where: v = flow velocity, m/s
C = hydraulic coefficient
R = hydraulic radius, m

The varying roughness coefficient is often reported in a graphical format. However, this relationship can also be described in equation form. A fourth order polynomial approximation of Camp's varying roughness coefficient is provided in Equation (5):

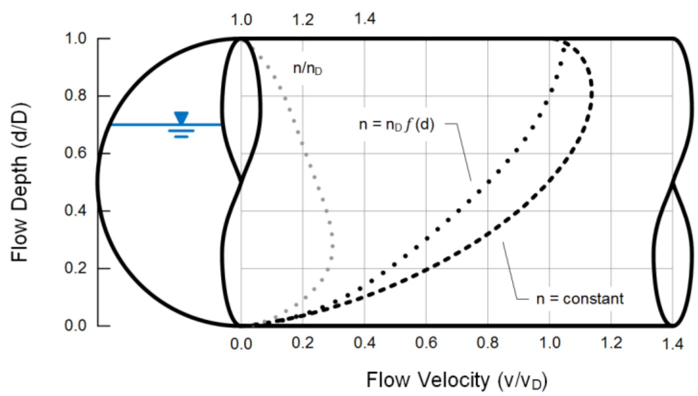
$$f(d) = 1.04 + 2.30\left(\frac{d}{D}\right) - 6.86\left(\frac{d}{D}\right)^2 + 7.79\left(\frac{d}{D}\right)^3 - 3.27\left(\frac{d}{D}\right)^4 \quad (5)$$

where: d = flow depth, mm or m
D = diameter, mm or m

Other equations have also been reported in the literature by various researchers, including Zaghloul, Wong and Zhou, and Akgiray.[5, 6, 7, 8] These equations are mathematically interchangeable with Equation (5) in subsequent discussions.

The relationship between flow depth and velocity described by the Manning Equation for a circular sewer is depicted in Figure 1 as a *pipe curve* and provides a convenient reference to evaluate flow monitor data. The relationships described using a constant roughness coefficient (- - -) and a varying roughness coefficient (· · ·) are provided for comparison.

FIGURE 1: Hydraulic Relationship of the Manning Equation

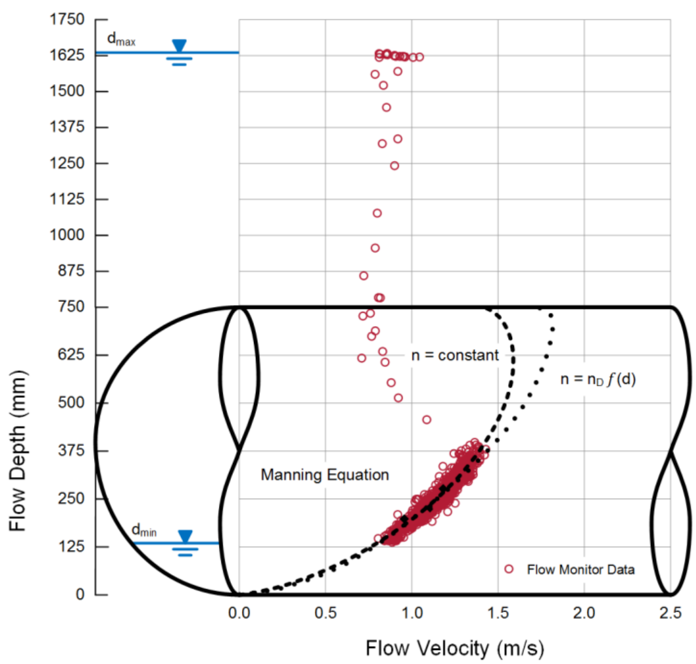


Manning Methods

The Manning Equation is also used to describe the performance of existing sewers by evaluating flow monitor data on a scattergraph, as shown in Figure 2. The Manning Equation is used to generate a pipe curve which is then compared to actual flow monitor data (○). This data may agree or disagree with the Manning Equation, depending on actual conditions at the monitoring location. In either case, important information can be learned about the performance of a sewer and its effect on sewer capacity.[9]

For example, the flow monitor data shown in Figure 2 indicate that this sewer operates as expected up to a flow depth of about 375 mm. However, as backwater conditions develop, flow conditions become deeper and slower and are revealed on the scattergraph as a departure from the pipe curve, resulting in surcharge and overflow conditions at a much lower capacity than expected.[10] Three manual confirmations (◆) are also shown and provide a means to evaluate the accuracy of the flow monitor.

FIGURE 2: Scattergraph of Flow Depth and Velocity Data

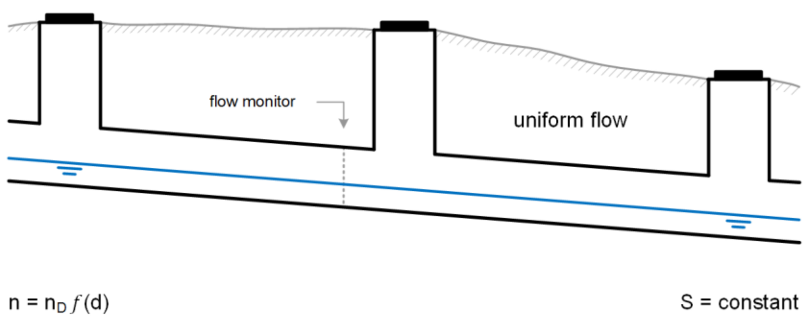


The Manning Equation is an important component of the scattergraph and can be applied using three different methods, defined as the *Design Method*, the *Lanfear-Coll Method*, and the *Stevens-Schutzbach Method*. The Design Method uses the Manning Equation to describe a relationship between flow depth and velocity using a specified roughness coefficient and pipe slope. This relationship is then compared with actual flow monitor data. The Lanfear-Coll Method and the Stevens-Schutzbach Method use curve fitting techniques to correlate the Manning Equation directly to such data. These methods have been previously reported using a constant roughness coefficient.[2] Modifications are presented in the following sections to incorporate a varying roughness coefficient into these methods.

Design Method

The *Design Method* uses the Manning Equation with a specified roughness coefficient and pipe slope and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 3.

FIGURE 3: General Assumptions of the Design Method



The modified Design Method incorporates the Manning Equation as expressed in Equation (6) and the hydraulic radius as defined in Equation (7).

$$v_{DM} = \frac{C_{DM}}{f(d)} R_{DM}^{2/3} \tag{6}$$

$$R_{DM} = \frac{A}{P} \tag{7}$$

where:
- v_{DM} = flow velocity, m/s
- C_{DM} = hydraulic coefficient
- R_{DM} = hydraulic radius, m
- A = wetted area, m^2
- P = wetted perimeter, m

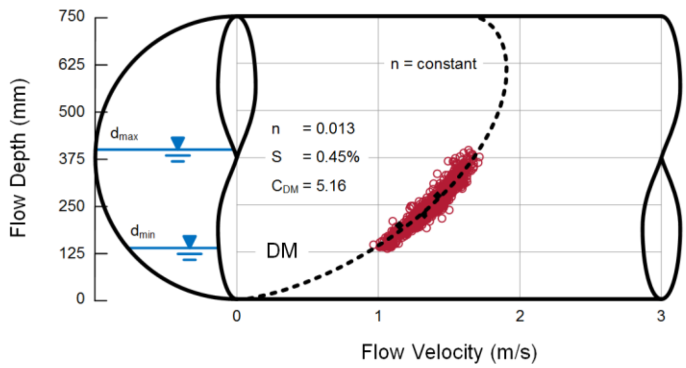
The roughness coefficient and the pipe slope are specified based on design assumptions, as-built documentation, or field observations and are used to calculate the hydraulic coefficient as shown in Equation (8).

$$C_{DM} = \frac{1}{n_D} S^{1/2} \tag{8}$$

where:
- C_{DM} = hydraulic coefficient
- n_D = roughness coefficient at d = D
- S = pipe slope

The Design Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 < d \leq D$. The application of the Design Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE Flow monitor data are obtained from a 750-mm sewer, as shown in the scattergraph below. A pipe curve has been previously constructed using the Design Method with a constant roughness coefficient, and the roughness coefficient (n) and pipe slope (S) are provided below.[2]



Use the Design Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare the result to the full-pipe capacity determined with a constant roughness coefficient.

Solution: Calculate the roughness coefficient and hydraulic coefficient

(a) Calculate n_D.

Based on the pipe curve using a constant roughness coefficient, assume that n = 0.013 within the range of observed data. Simply further, and assume n = 0.013 for d = 250 mm. Calculate f(d) for d = 250 mm. using Equation (5), then calculate n_D using Equation (2).

$n_D = 0.010$

(b) Calculate C_{DM} assuming $n_D = 0.010$ and S = 0.45%.

$C_{DM} = 6.67$

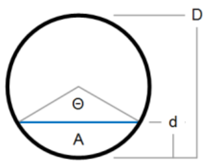
Complete calculations are available in a spreadsheet that accompanies this technical paper.

EXAMPLE

Solution: Construct pipe curve and estimate sewer capacity

(c) Calculate v_{DM} for $0 < d \le D$.

For a circular sewer,[11]



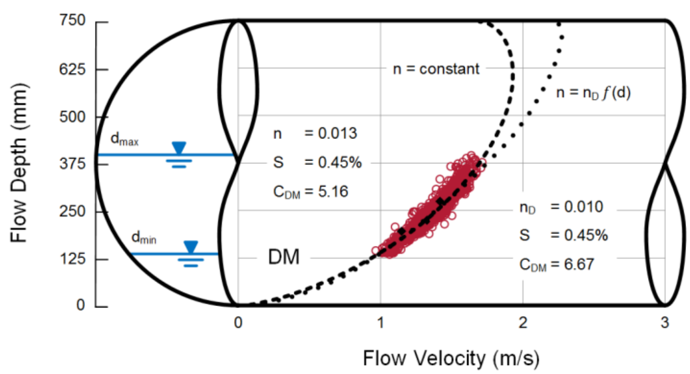
$\Theta = 2\cos^{-1}(1 - 2d/D)$

$A = (D^2/8)(\Theta - \sin\Theta)$

$P = D\Theta/2$

d	Θ	A	P	R_{DM}	f(d)	$R_{LC}^{2/3}/f(d)$	v_{DM}
mm	°	m^2	m	m		$m^{2/3}$	m/s
0	0	0.000	0.000	—	1.04	—	—
125	96	0.048	0.631	0.077	1.27	0.143	0.95
250	141	0.129	0.923	0.140	1.29	0.208	1.39
375	180	0.221	1.178	0.188	1.24	0.263	1.76
500	219	0.313	1.433	0.218	1.19	0.306	2.04
625	264	0.393	1.725	0.228	1.12	0.332	2.21
750	360	0.442	2.356	0.188	1.00	0.328	2.19

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation using the Design Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

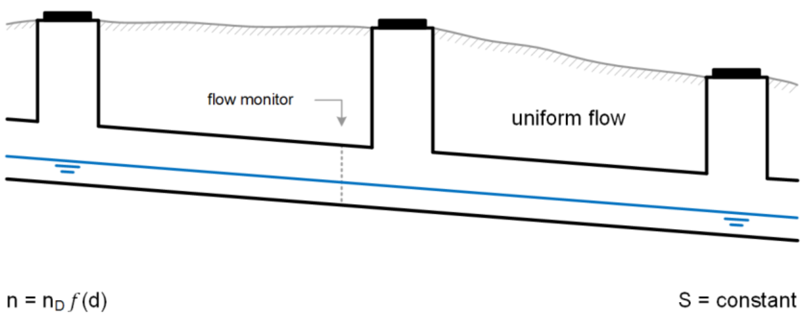
(d) Calculate Q_{DM} for d = D.

The full-pipe capacity is calculated using the Continuity Equation, $Q_{DM} = Av_{DM}$. Therefore, Q_{DM} = 0.442 m^2 x 2.19 m/s = 0.965 m^3/s or 965 L/s, about 30% greater than the corresponding value determined using a constant roughness coefficient.[2]

Lanfear-Coll Method

The Lanfear-Coll Method uses a curve fitting technique to fit the Manning Equation to flow monitor data and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 4.

FIGURE 4: General Assumptions of the Lanfear-Coll Method



The modified Lanfear-Coll method is applicable to flow monitor data obtained under uniform flow conditions and incorporates the Manning Equation as expressed in Equation (9) and the hydraulic radius as defined in Equation (10).

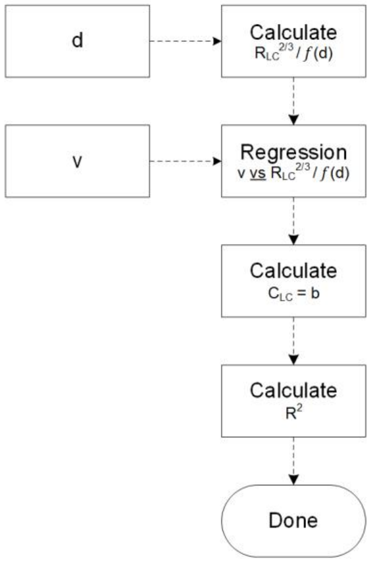
$$v_{LC} = \frac{C_{LC}}{f(d)} R_{LC}^{2/3} \tag{9}$$

$$R_{LC} = \frac{A}{P} \tag{10}$$

where:
- v_{LC} = flow velocity, m/s
- C_{LC} = hydraulic coefficient
- R_{LC} = hydraulic radius, m
- A = wetted area, m^2
- P = wetted perimeter, m

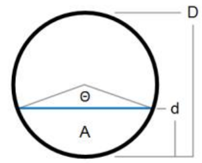
This method provides an implicit solution to the Manning Equation and requires no direct knowledge of the roughness coefficient or the slope of the energy gradient. Flow depth and velocity data are used to calculate the hydraulic coefficient based on a least-squares regression of Equation (9) using a varying roughness coefficient, as described in Figure 5. Regression results are characterized using the coefficient of determination.[12]

FIGURE 5: Regression Using the Lanfear-Coll Method



For a circular sewer,

$$\Theta = 2\cos^{-1}(1 - 2d/D)$$
$$A = (D^2/8)(\Theta - \sin\Theta)$$
$$P = D\Theta/2$$

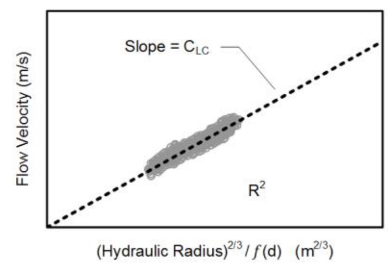


$$R_{LC} = \frac{A}{P}$$

Restate Equation (9) as y = a + bx using direct substitution, where:

$$x = R_{LC}^{2/3} / f(d)$$
$$y = v_{LC}$$
$$a = 0$$
$$b = C_{LC}$$

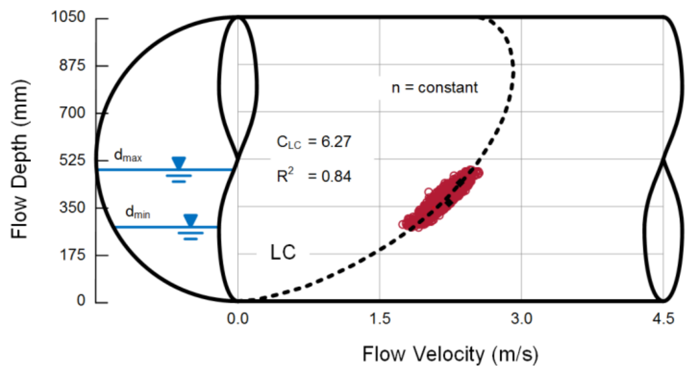
Perform least squares regression.



The Lanfear-Coll Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 < d \leq D$. The application of the Lanfear-Coll Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE

Flow monitor data are obtained from a 1050-mm sewer, as shown in the scattergraph below. Tabular data are provided on the following page. A pipe curve has been previously constructed using the Lanfear-Coll Method with a constant roughness coefficient.[2]



Use the Lanfear-Coll Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare the result to the full-pipe capacity determined with a constant roughness coefficient.

EXAMPLE

Solution: Calculate the hydraulic coefficient

(a) Calculate $R_{LC}^{2/3} / f(d)$.

date	time	d	v	Θ	A	P	R_{LC}	f(d)	$R_{LC}^{2/3} / f(d)$
mm/dd	hh:mm	mm	m/s	°	m^2	m	m		$m^{2/3}$
11/01	00:00	355	2.19	142	0.258	1.304	0.198	1.29	0.263
11/01	00:15	356	2.26	143	0.259	1.306	0.198	1.29	0.263
11/01	00:30	348	2.17	141	0.251	1.289	0.195	1.29	0.260
11/01	00:45	345	2.18	140	0.248	1.282	0.193	1.29	0.259
11/01	01:00	336	2.10	138	0.239	1.262	0.189	1.29	0.254
11/01	01:15	334	2.13	137	0.237	1.259	0.188	1.29	0.254
11/01	01:30	334	2.08	137	0.237	1.258	0.188	1.29	0.254
11/01	01:45	330	2.05	136	0.233	1.251	0.187	1.30	0.252
11/01	02:00	325	2.05	135	0.229	1.240	0.184	1.30	0.250
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
11/30	23:45	397	2.20	152	0.300	1.391	0.216	1.28	0.280
			v_{avg} ← 2.19						

(b) Calculate C_{LC} and R^2 based on a least squares regression.

date	time	x	y	xy	x^2	v_{LC}	$(v_{LC} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$m^{2/3}$	m/s	$m^{5/3}/s$	$m^{4/3}$	m/s	$(m/s)^2$	$(m/s)^2$
11/01	00:00	0.263	2.19	0.575	0.069	2.12	0.005	0.000
11/01	00:15	0.263	2.26	0.594	0.069	2.12	0.018	0.005
11/01	00:30	0.260	2.17	0.563	0.068	2.09	0.005	0.000
11/01	00:45	0.259	2.18	0.563	0.067	2.08	0.009	0.000
11/01	01:00	0.254	2.10	0.534	0.065	2.05	0.002	0.007
11/01	01:15	0.254	2.13	0.542	0.064	2.05	0.008	0.003
11/01	01:30	0.254	2.08	0.527	0.064	2.04	0.001	0.011
11/01	01:45	0.252	2.05	0.516	0.064	2.03	0.000	0.020
11/01	02:00	0.250	2.05	0.511	0.062	2.01	0.001	0.020
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
11/30	23:45	0.280	2.20	0.617	0.079	2.26	0.003	0.000
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,880 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{LC} = \sum xy \Big/ \sum x^2$$

$$R^2 = 1 - \frac{SSE}{SYY}$$

Based on the regression results, C_{LC} = 8.06 and R^2 = 0.82.

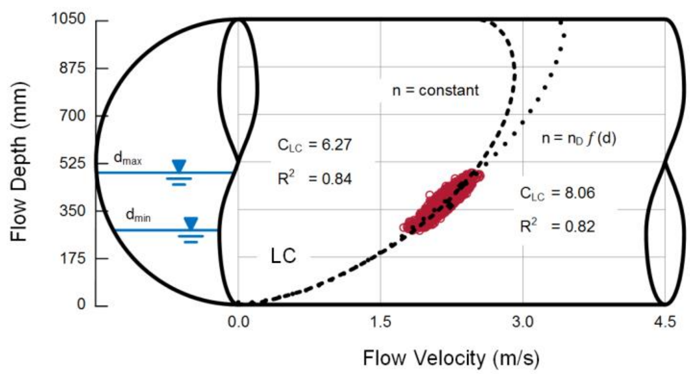
EXAMPLE

Solution: Construct pipe curve and estimate sewer capacity

(c) Calculate v_{LC} for $0 < d \leq D$.

d	Θ	A	P	R_{LC}	$f(d)$	$R_{LC}^{2/3}/f(d)$	v_{LC}
mm	°	m^2	m	m		$m^{2/3}$	m/s
0	0	0.000	0.000	—	1.04	—	—
175	96	0.095	0.883	0.107	1.27	0.178	1.44
350	141	0.253	1.293	0.195	1.29	0.261	2.10
525	180	0.433	1.649	0.263	1.24	0.329	2.65
700	219	0.613	2.006	0.306	1.19	0.382	3.08
875	264	0.771	2.416	0.319	1.12	0.416	3.35
1050	360	0.866	3.299	0.263	1.00	0.410	3.30

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation fitted to observed flow depth and velocity data using the Lanfear-Coll Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

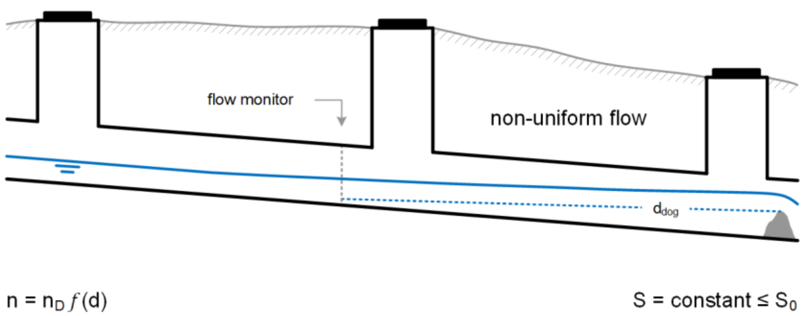
(d) Calculate Q_{LC} for d = D.

The full-pipe capacity is calculated using the Continuity Equation, $Q_{LC} = Av_{LC}$. Therefore, Q_{LC} = 0.866 m^2 x 3.30 m/s = 2.861 m^3/s or 2861 L/s, about 30% greater than the corresponding value determined using a constant roughness coefficient.[2]

Stevens-Schutzbach Method

The Stevens-Schutzbach Method uses an iterative curve fitting technique to fit the Manning Equation to flow monitor data and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 6.

FIGURE 6: General Assumptions of the Stevens-Schutzbach Method



This method is applicable to flow monitor data obtained under uniform flow conditions or non-uniform flow conditions resulting from a variety of downstream obstructions, or *dead dogs*, where the slope of the energy gradient is less than the pipe slope. Examples include offset joints, debris, and other related conditions. The modified Stevens-Schutzbach Method incorporates the Manning Equation as expressed in Equation (11) and the hydraulic radius as defined in Equation (12).

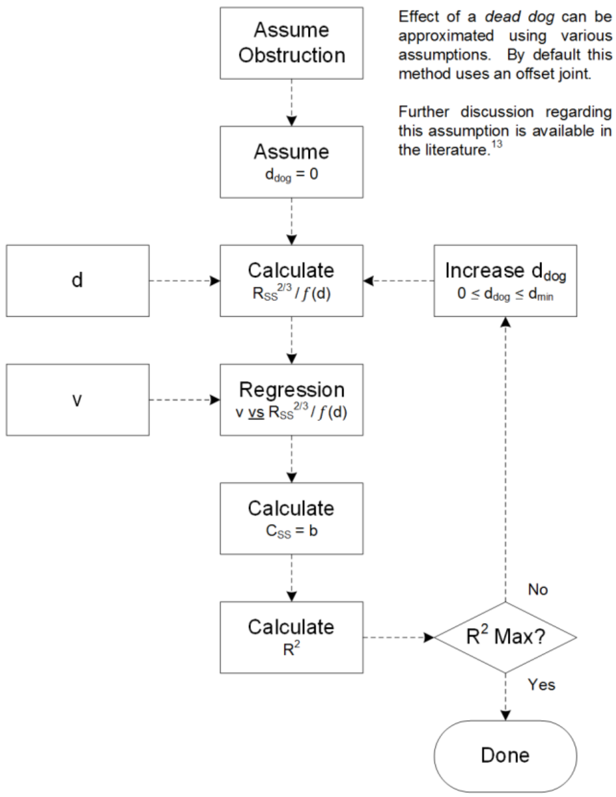
$$v_{SS} = \frac{C_{SS}}{f(d)} R_{SS}^{2/3} \tag{11}$$

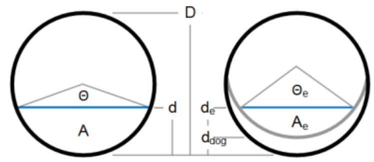
$$R_{SS} = \frac{A_e}{P} \tag{12}$$

where:
v_{SS} = flow velocity, m/s
C_{SS} = hydraulic coefficient
R_{SS} = hydraulic radius, m
A_e = effective wetted area, m^2
P = wetted perimeter, m

Note that the definition of the hydraulic radius is modified from the traditional definition and requires certain assumptions regarding the shape and magnitude of the *dead dog*. Based on these assumptions, flow depth and velocity data are used to calculate the hydraulic coefficient based on an iterative least-squares regression method using a varying roughness coefficient, as described in Figure 7. The magnitude of the *dead dog* (d_{dog}) is varied in successive iterations until the coefficient of determination is maximized.

FIGURE 7: Regression Using the Stevens-Schutzbach Method





For a circular sewer,

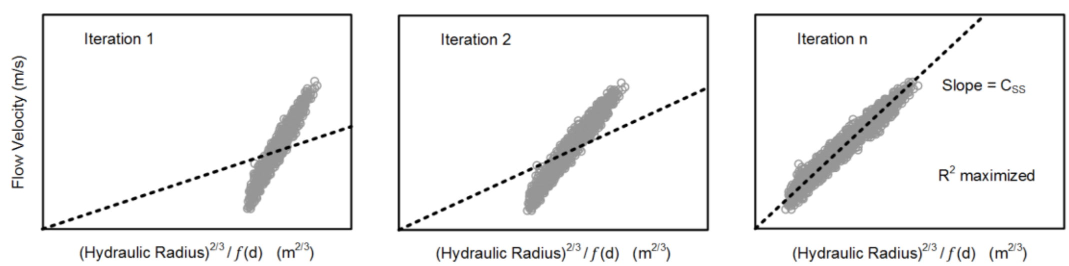
$$\Theta = 2\cos^{-1}(1 - 2d/D)$$
$$A = (D^2/8)(\Theta - \sin \Theta)$$
$$P = D\Theta / 2$$

$$d_e = d - d_{dog}$$
$$\Theta_e = 2\cos^{-1}(1 - 2d_e/D)$$
$$A_e = (D^2/8)(\Theta_e - \sin \Theta_e)$$
$$R_{SS} = \frac{A_e}{P}$$

Restate Equation (11) as $y = a + bx$ using direct substitution, where:

$$x = R_{SS}^{2/3} / f(d)$$
$$y = v_{SS}$$
$$a = 0$$
$$b = C_{SS}$$

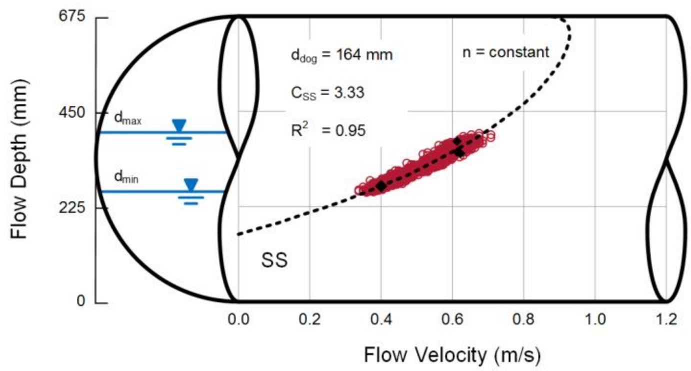
Perform iterative least squares regression.



The Stevens-Schutzbach Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 < d \le D$. The application of the Stevens-Schutzbach Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE

Flow monitor data are obtained from a 675-mm sewer, as shown in the scattergraph below. Tabular data are provided on the following page. A pipe curve has been previously constructed using the Stevens-Schutzbach Method with a constant roughness coefficient.[2]



Use the Stevens-Schutzbach Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare this result to the full-pipe capacity determined with a constant roughness coefficient.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 1

(a) Assume d_{dog} = 0 mm. Calculate $R_{SS}^{2/3} / f(d)$.

date	time	d	v	d_e	Θ_e	A_e	Θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	mm	m/s	mm	°	m^2	°	m	m		$m^{2/3}$
08/01	00:00	364	0.64	364	189	0.197	189	1.114	0.177	1.23	0.256
08/01	00:15	358	0.62	358	187	0.193	187	1.101	0.175	1.23	0.253
08/01	00:30	353	0.61	353	185	0.190	185	1.092	0.174	1.24	0.252
08/01	00:45	351	0.60	351	185	0.188	185	1.087	0.173	1.24	0.251
08/01	01:00	342	0.61	342	182	0.182	182	1.070	0.170	1.24	0.247
08/01	01:15	334	0.59	334	179	0.176	179	1.053	0.168	1.25	0.244
08/01	01:30	328	0.56	328	177	0.173	177	1.042	0.166	1.25	0.242
08/01	01:45	331	0.57	331	178	0.175	178	1.048	0.167	1.25	0.243
08/01	02:00	327	0.55	327	176	0.172	176	1.040	0.165	1.25	0.241
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	360	0.63	360	188	0.194	188	1.106	0.176	1.23	0.255
	263 →	d_{min}	v_{avg}	← 0.57							

(b) Calculate C_{SS} and R^2 based on a least squares regression.

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$m^{2/3}$	m/s	$m^{5/3}/s$	$m^{4/3}$	m/s	$(m/s)^2$	$(m/s)^2$
08/01	00:00	0.256	0.64	0.165	0.066	0.59	0.003	0.006
08/01	00:15	0.253	0.62	0.157	0.064	0.58	0.001	0.003
08/01	00:30	0.252	0.61	0.153	0.063	0.58	0.001	0.002
08/01	00:45	0.251	0.60	0.150	0.063	0.58	0.000	0.001
08/01	01:00	0.247	0.61	0.150	0.061	0.57	0.001	0.002
08/01	01:15	0.244	0.59	0.143	0.059	0.56	0.001	0.000
08/01	01:30	0.242	0.56	0.136	0.058	0.56	0.000	0.000
08/01	01:45	0.243	0.57	0.139	0.059	0.56	0.000	0.000
08/01	02:00	0.241	0.55	0.132	0.058	0.55	0.000	0.000
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.255	0.63	0.161	0.065	0.59	0.002	0.004
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \sum xy \;/\; \sum x^2$$

$$R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.30 and R^2 = 0.61. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 2

(a) Assume d_{dog} = 25 mm. Calculate $R_{SS}^{2/3} / f(d)$.

date	time	d	v	d_e	Θ_e	A_e	Θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	mm	m/s	mm	°	m^2	°	m	m		$m^{2/3}$
08/01	00:00	364	0.64	339	181	0.180	189	1.114	0.162	1.23	0.241
08/01	00:15	358	0.62	333	178	0.176	187	1.101	0.160	1.23	0.238
08/01	00:30	353	0.61	328	177	0.173	185	1.092	0.158	1.24	0.237
08/01	00:45	351	0.60	326	176	0.171	185	1.087	0.157	1.24	0.236
08/01	01:00	342	0.61	317	173	0.165	182	1.070	0.155	1.24	0.232
08/01	01:15	334	0.59	309	170	0.160	179	1.053	0.152	1.25	0.228
08/01	01:30	328	0.56	303	168	0.156	177	1.042	0.150	1.25	0.226
08/01	01:45	331	0.57	306	169	0.158	178	1.048	0.151	1.25	0.227
08/01	02:00	327	0.55	302	168	0.155	176	1.040	0.149	1.25	0.225
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	360	0.63	335	179	0.178	188	1.106	0.160	1.23	0.240
	263 →	d_{min}	v_{avg}	← 0.57							

(b) Calculate C_{SS} and R^2 based on a least squares regression.

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$m^{2/3}$	m/s	$m^{5/3}/s$	$m^{4/3}$	m/s	$(m/s)^2$	$(m/s)^2$
08/01	00:00	0.241	0.64	0.155	0.058	0.59	0.003	0.006
08/01	00:15	0.238	0.62	0.148	0.057	0.59	0.001	0.003
08/01	00:30	0.237	0.61	0.144	0.056	0.58	0.001	0.002
08/01	00:45	0.236	0.60	0.141	0.055	0.58	0.000	0.001
08/01	01:00	0.232	0.61	0.141	0.054	0.57	0.001	0.002
08/01	01:15	0.228	0.59	0.133	0.052	0.56	0.001	0.000
08/01	01:30	0.226	0.56	0.127	0.051	0.55	0.000	0.000
08/01	01:45	0.227	0.57	0.130	0.051	0.56	0.000	0.000
08/01	02:00	0.225	0.55	0.123	0.051	0.55	0.000	0.000
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.240	0.63	0.151	0.057	0.59	0.002	0.004
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \sum xy \Big/ \sum x^2 \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.46 and R^2 = 0.67. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 3

(a) Assume d_{dog} = 50 mm. Calculate $R_{SS}^{2/3} / f(d)$.

date	time	d	v	d_e	Θ_e	A_e	Θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	mm	m/s	mm	°	m^2	°	m	m		$m^{2/3}$
08/01	00:00	364	0.64	314	172	0.163	189	1.114	0.147	1.23	0.226
08/01	00:15	358	0.62	308	170	0.159	187	1.101	0.144	1.23	0.223
08/01	00:30	353	0.61	303	168	0.156	185	1.092	0.143	1.24	0.221
08/01	00:45	351	0.60	301	168	0.154	185	1.087	0.142	1.24	0.220
08/01	01:00	342	0.61	292	165	0.149	182	1.070	0.139	1.24	0.216
08/01	01:15	334	0.59	284	162	0.143	179	1.053	0.136	1.25	0.212
08/01	01:30	328	0.56	278	160	0.139	177	1.042	0.134	1.25	0.209
08/01	01:45	331	0.57	281	161	0.141	178	1.048	0.135	1.25	0.211
08/01	02:00	327	0.55	277	159	0.138	176	1.040	0.133	1.25	0.209
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	360	0.63	310	171	0.161	188	1.106	0.145	1.23	0.224
	263 →	d_{min}	v_{avg}	← 0.57							

(b) Calculate C_{SS} and R^2 based on a least squares regression.

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$m^{2/3}$	m/s	$m^{5/3}/s$	$m^{4/3}$	m/s	$(m/s)^2$	$(m/s)^2$
08/01	00:00	0.226	0.64	0.145	0.051	0.60	0.002	0.006
08/01	00:15	0.223	0.62	0.138	0.050	0.59	0.001	0.003
08/01	00:30	0.221	0.61	0.134	0.049	0.58	0.001	0.002
08/01	00:45	0.220	0.60	0.131	0.048	0.58	0.000	0.001
08/01	01:00	0.216	0.61	0.131	0.047	0.57	0.001	0.002
08/01	01:15	0.212	0.59	0.124	0.045	0.56	0.001	0.000
08/01	01:30	0.209	0.56	0.117	0.044	0.55	0.000	0.000
08/01	01:45	0.211	0.57	0.121	0.044	0.56	0.000	0.000
08/01	02:00	0.209	0.55	0.114	0.044	0.55	0.000	0.000
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.224	0.63	0.141	0.050	0.59	0.002	0.004
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \sum xy \;/\; \sum x^2$$

$$R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.64 and R^2 = 0.74. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration n

(a) Assume d_{dog} = 153 mm. Calculate $R_{SS}^{2/3} / f(d)$.

date	time	d	v	d_e	Θ_e	A_e	Θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	mm	m/s	mm	°	m^2	°	m	m		$m^{2/3}$
08/01	00:00	364	0.64	212	136	0.096	189	1.114	0.086	1.23	0.159
08/01	00:15	358	0.62	205	134	0.092	187	1.101	0.083	1.23	0.155
08/01	00:30	353	0.61	201	132	0.089	185	1.092	0.082	1.24	0.152
08/01	00:45	351	0.60	198	131	0.088	185	1.087	0.081	1.24	0.151
08/01	01:00	342	0.61	190	128	0.082	182	1.070	0.077	1.24	0.146
08/01	01:15	334	0.59	181	125	0.077	179	1.053	0.073	1.25	0.141
08/01	01:30	328	0.56	176	123	0.074	177	1.042	0.071	1.25	0.137
08/01	01:45	331	0.57	179	124	0.076	178	1.048	0.072	1.25	0.139
08/01	02:00	327	0.55	174	122	0.073	176	1.040	0.071	1.25	0.137
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	360	0.63	208	135	0.094	188	1.106	0.085	1.23	0.156
	263 →	d_{min}	v_{avg}	← 0.57							

(b) Calculate C_{SS} and R^2 based on a least squares regression.

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$m^{2/3}$	m/s	$m^{5/3}/s$	$m^{4/3}$	m/s	$(m/s)^2$	$(m/s)^2$
08/01	00:00	0.159	0.64	0.102	0.025	0.62	0.001	0.006
08/01	00:15	0.155	0.62	0.096	0.024	0.60	0.000	0.003
08/01	00:30	0.152	0.61	0.092	0.023	0.59	0.000	0.002
08/01	00:45	0.151	0.60	0.090	0.023	0.59	0.000	0.001
08/01	01:00	0.146	0.61	0.088	0.021	0.57	0.002	0.002
08/01	01:15	0.141	0.59	0.082	0.020	0.55	0.001	0.000
08/01	01:30	0.137	0.56	0.077	0.019	0.53	0.001	0.000
08/01	01:45	0.139	0.57	0.080	0.019	0.54	0.001	0.000
08/01	02:00	0.137	0.55	0.075	0.019	0.53	0.000	0.000
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.156	0.63	0.099	0.024	0.61	0.001	0.004
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \sum xy \Big/ \sum x^2 \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 3.89 and R^2 = 0.95. R^2 is maximized.

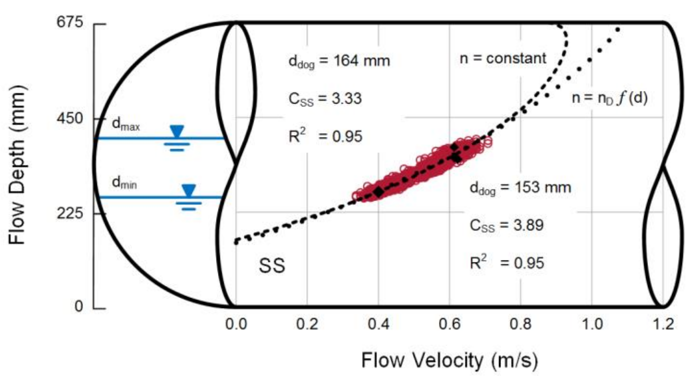
EXAMPLE

Solution: Construct pipe curve and estimate sewer capacity

(c) Calculate v_{SS} for $0 < d \leq D$.

d	d_e	Θ_e	A_e	Θ	A	P	R_{SS}	f(d)	$R_{SS}^{2/3}/f(d)$	v_{SS}
mm	mm	°	m^2	°	m^2	m	m		$m^{2/3}$	m/s
0	0	0	0.000	0	0.000	0.000	—	1.04	—	—
75	0	0	0.000	78	0.022	0.459	0.000	1.22	0.000	0.00
150	0	0	0.000	113	0.059	0.663	0.000	1.29	0.000	0.00
225	72	76	0.021	141	0.104	0.831	0.025	1.29	0.066	0.26
300	147	111	0.058	167	0.154	0.985	0.059	1.26	0.119	0.46
375	222	140	0.103	193	0.204	1.135	0.090	1.22	0.165	0.64
450	297	166	0.152	219	0.253	1.290	0.118	1.19	0.202	0.79
525	372	192	0.202	247	0.299	1.458	0.139	1.15	0.234	0.91
600	447	218	0.252	282	0.336	1.662	0.151	1.09	0.260	1.01
675	552	246	0.297	360	0.358	2.121	0.140	1.00	0.270	1.05

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation fitted to observed flow depth and velocity data using the Stevens-Schutzbach Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

(d) Calculate Q_{SS} for d = D.

The full-pipe capacity is calculated using the Continuity Equation, $Q_{SS} = Av_{SS}$. Therefore, Q_{SS} = 0.358 m^2 x 1.05 m/s = 0.375 m^3/s or 375 L/s, about 20% greater than the corresponding value determined with a constant roughness coefficient.[2]

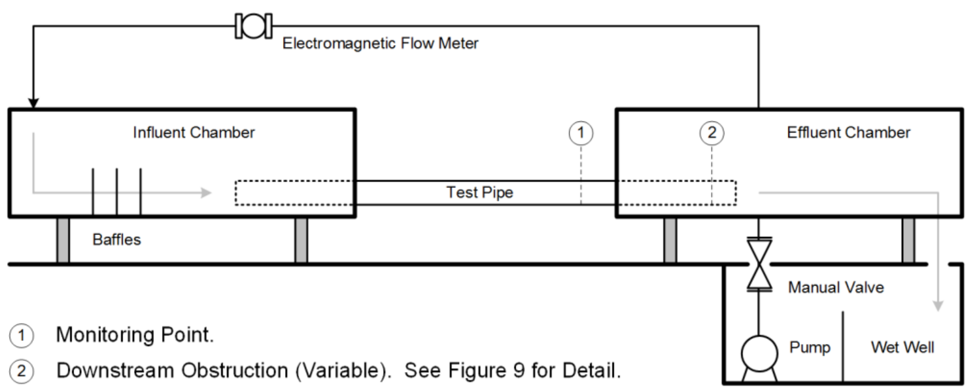
Laboratory Investigation

Laboratory investigations were previously reported to demonstrate the performance of the Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method under controlled conditions.[2] The results of these investigations can also be used to compare the use of a constant and varying roughness coefficient.

Equipment and Methodology

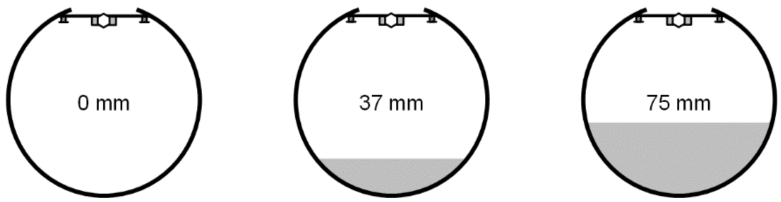
The laboratory equipment used during this investigation was designed and configured to simulate hydraulic conditions encountered in the urban sewer environment. The general arrangement of this equipment is provided in Figure 8.

FIGURE 8: Laboratory General Arrangement



A pump provides flow through a 150-mm PVC force main to an influent chamber. A manual valve regulates the pump, and an electromagnetic flow meter measures the pump discharge. Flow passes through three consecutive baffles within the influent chamber, minimizing surface disturbances before entering a 200-mm PVC test pipe. Uniform and non-uniform flow conditions are observed and measured at a monitoring point located within the test pipe. Flow conditions are controlled using one of three obstructions of known depth, as depicted in Figure 9, positioned a fixed distance downstream from the monitoring point. Following discharge from the test pipe to an effluent chamber, the flow is returned to a wet well for re-circulation by the pump.

FIGURE 9: Downstream Obstructions for Laboratory Investigation

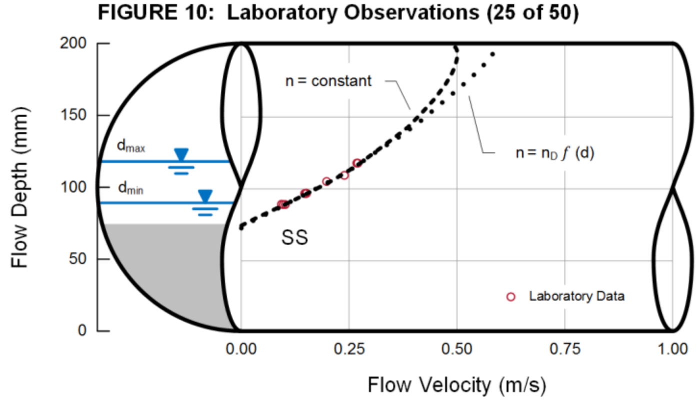


After placing an obstruction within the test pipe, the pump is activated, and flow is introduced into the system. Once the system has reached equilibrium, flow depth and quantity measurements are obtained at five consecutive one-minute intervals. Flow depth is measured in the test pipe with a stainless steel ruler, and flow quantity is measured in the force main with the electromagnetic flow meter. These measurements are then used to calculate flow velocity in the test pipe using the Continuity Equation. A total of 50 flow depth and quantity measurements were obtained at a variety of pump settings.

Results and Discussion

Flow depth and velocity data obtained using the 75-mm downstream obstruction are plotted on a scattergraph and evaluated with respect to the Manning Equation using the Stevens-Schutzbach Method. This method is applied to the first 25 laboratory observations using both a constant and varying roughness coefficient as shown in Figure 10.

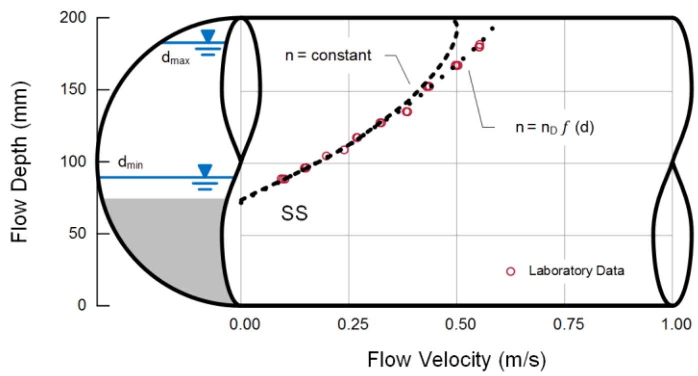
FIGURE 10: Laboratory Observations (25 of 50)



Note that these observations are effectively described using either a constant or varying roughness coefficient. The sum of the squared error (SSE) for the Stevens-Schutzbach Method using a constant and a varying roughness coefficient is 0.001 $(m/s)^2$, but which assumption provides the best estimate of actual sewer capacity under full-pipe conditions? For a varying roughness coefficient, the projected full-pipe velocity is 0.59 m/s, with an estimated sewer capacity of 18 L/s – about 20% greater than the corresponding value determined using a constant roughness coefficient.

To further test the two assumptions, the remaining 25 laboratory observations are added to the scattergraph and compared with the existing pipe curves from Figure 10, as shown in Figure 11.

FIGURE 11: Laboratory Observations (50 of 50)



The SSE for these observations is 0.030 $(m/s)^2$ using a constant roughness coefficient, while the SSE is 0.002 $(m/s)^2$ using a varying roughness coefficient. The SSE for the constant roughness coefficient is over 10 times greater than the SSE for the varying roughness coefficient. These results indicate that the varying roughness coefficient provides a more accurate projection of sewer capacity than the constant roughness coefficient under these test conditions.

Conclusion

The scattergraph is a graphical tool that provides insight into sewer performance through a simple and intuitive display of flow monitor data. The Manning Equation is an important component of the scattergraph and can be applied using a variety of methods. The Design Method uses the Manning Equation to describe a relationship between flow depth and velocity using a specified roughness coefficient and pipe slope. This relationship is then compared with actual flow monitor data. The Lanfear-Coll Method and the Stevens-Schutzbach Method use curve fitting techniques to correlate the Manning Equation directly to such data. Modifications are presented to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%. Laboratory results indicate that the use of a varying roughness coefficient provides a more accurate determination of sewer capacity.

Symbols and Notation

The following symbols and notation are used in this paper:

VARIABLES

d = flow depth, mm or m
v = flow velocity, m/s
Q = flow rate, m^3/s or L/s
n = roughness coefficient
R = hydraulic radius, m
S = slope of the energy gradient
C = hydraulic coefficient
D = diameter, mm or m
A = wetted area, m^2
P = wetted perimeter, m
R^2 = coefficient of determination

SUBSCRIPTS

$_{DM}$ = Design Method
$_{LC}$ = Lanfear-Coll Method
$_{SS}$ = Stevens-Schutzbach Method
$_{dog}$ = *dead dog*
$_0$ = specified
$_e$ = effective
$_{avg}$ = average
$_{min}$ = minimum

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Enfinger, K.L. and Schutzbach, J.S. (2005). "Scattergraph Principles and Practice – Camp's Varying Roughness Coefficient Applied to the Manning Equation," *Proceedings of the Pipeline Division Specialty Conference*; Houston, TX; American Society of Civil Engineers: Reston, VA.