
Working with Combined Sewers

Geometric and Hydraulic Elements of Egg-Shaped Sewers

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ABSTRACT

Wastewater professionals working with combined sewers often encounter a variety of non-circular sewer cross sections, including egg-shaped sewers. Equations are developed here to calculate the wetted area, wetted perimeter, and surface width for egg-shaped sewers flowing partially full and support a variety of hydraulic calculations. These equations supplant traditional graphical approaches that use geometric and hydraulic elements curves in favor of equations better suited to Smart Sewer applications. The equations are validated and provide a concise reference for those working with egg-shaped sewers in combined sewer systems.

KEY WORDS

Combined Sewer, Non-Circular Section, Egg-Shaped Sewer

Introduction

Combined sewers are designed and constructed with a wide variety of sewer cross sections. While the circular section is most common, other non-circular sections have been used over the years.¹ Working with combined sewers requires an understanding of the geometric and hydraulic elements of these non-circular sections when encountered.

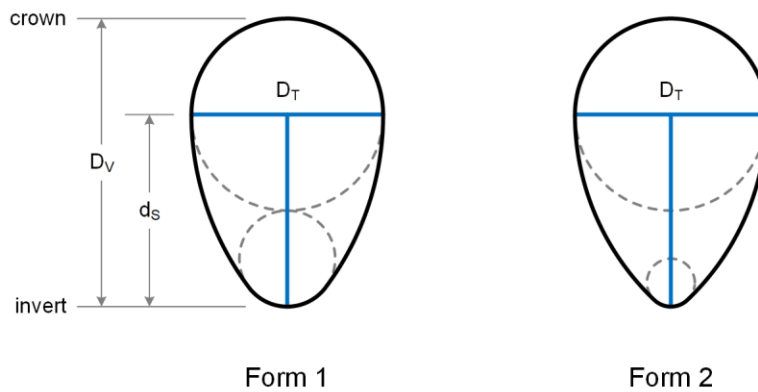
Geometric properties of non-circular sections are found in standard engineering references but are often presented only in graphical form for sewers flowing partially full and where the results are provided as a proportion of values for sewers flowing full.¹ While this approach was sufficient when these sewers were originally designed and constructed, it is insufficient for modern Smart Sewer applications where automated and precise computations are required.

The egg-shaped sewer section is the focus of this discussion. Descriptions of the most common egg-shaped sewer sections (Form 1 and Form 2) are provided, and historical perspectives regarding their development and application are discussed. The geometric elements of these egg-shaped sections are then defined, and equations for the wetted area, wetted perimeter, and surface width for sewers flowing partially full are provided. The result is a concise reference for wastewater professionals working with egg-shaped sewers in combined sewer systems.

Description

While various egg-shaped sewers have been developed and used over the years, two versions – Form 1 and Form 2 – are most commonly encountered as shown in Figure 1. The geometry of these egg-shaped sewers begins with the *transverse diameter* (D_T), and other relevant dimensions – including the vertical diameter (D_V) – are each defined as a function of D_T .

FIGURE 1: Various Egg-Shaped Sewer Cross-Sections



The anatomy of the Form 1 egg-shaped sewer is shown in Figure 2, and the anatomy of the Form 2 egg-shaped sewer is shown in Figure 3.

FIGURE 2: Anatomy of an Egg-Shaped Sewer – Form 1

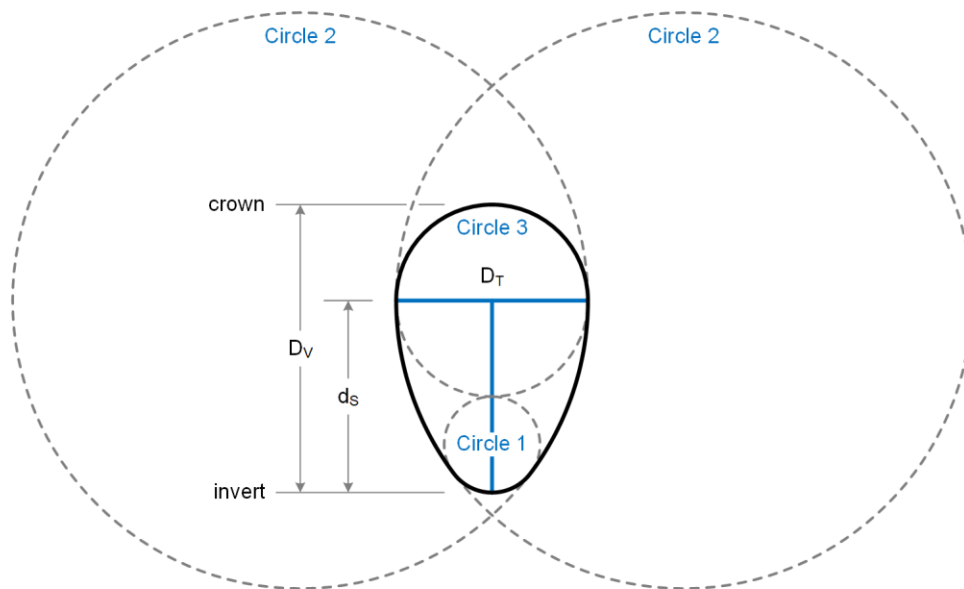
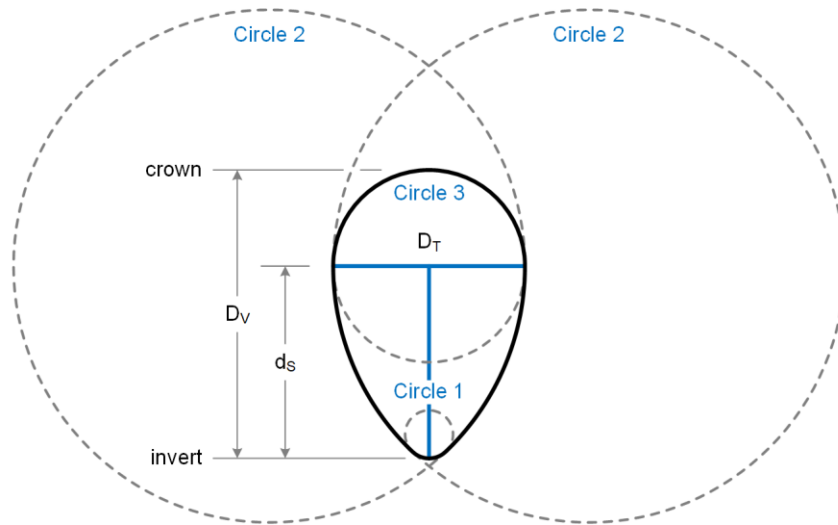


FIGURE 3: Anatomy of an Egg-Shaped Sewer – Form 2



Both Form 1 and Form 2 egg-shaped sewers are compound cross-sections formed by several tangent circles. In both cases, Circle 1 forms the invert of the egg-shaped sewer, which is positioned at a specified distance (d_s) from the invert to the spring line. Circle 2 forms the sides, and Circle 3 forms the arch. Relative dimensions of each component of Form 1 and Form 2 egg-shaped sewers are provided in Table 1.

TABLE 1: Relative Component Dimensions of Various Egg-Shaped Sewers

Component	Form 1		Form 2	
	Diameter	Depth	Diameter	Depth
Circle 1	$D_1 = \frac{1}{2}D_T$	$d_1 = d$	$D_1 = \frac{1}{4}D_T$	$d_1 = d$
Circle 2	$D_2 = 3D_T$	$d_2 = d + \frac{1}{2}D_T$	$D_2 = \frac{8}{3}D_T$	$d_2 = d + \frac{1}{3}D_T$
Circle 3	$D_3 = D_T$	$d_3 = d - \frac{1}{2}D_T$	$D_3 = D_T$	$d_3 = d - \frac{1}{2}D_T$
Egg	$D_V = \frac{3}{2}D_T$	$d_s = D_T$	$D_V = \frac{3}{2}D_T$	$d_s = D_T$

The diameter (D_1, D_2, D_3) of each component circle is defined relative to D_T as described by Baldwin Latham, and the flow depth in each component circle (d_1, d_2, d_3) is derived relative to the flow depth (d) in the egg-shaped sewer.²

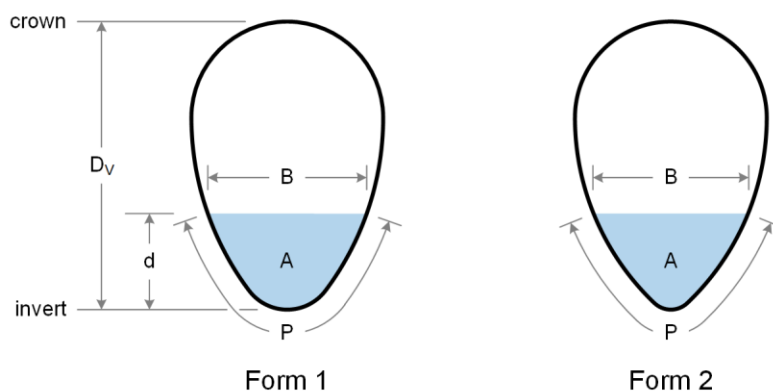
Historical Perspective

The egg-shaped sewer is a 19th Century innovation designed to enhance self-cleansing in combined sewers by channeling wastewater flows through a smaller cross-section during low flow conditions, resulting in a greater flow velocity when compared to a similarly sized circular sewer.² Two British engineers are most associated with its early design and use. John Roe (1795 – 1874) is believed to be the first to design them, and they were first constructed for the Holborn and Finsbury Commissioners of Sewers in Greater London.³ Unfortunately, details of his design and date of first use are not readily revealed in contemporary engineering literature. However, since the Commissioners did not appoint John Roe as Surveyor until 1830, it is probable that the first egg-shaped sewers appeared sometime after this date. While not the first to design them, John Phillips (1817 – 1897) is associated with several egg-shaped sewer designs that became most popular. Two of his designs are commonly referenced in contemporary engineering literature as the *old form* and the *new form*, while more recent engineering literature references them as Form 1 and Form 2.^{2,4}

Geometric Elements

Several *geometric elements* are needed for engineering calculations involving sewers flowing partially full and are a function of flow depth (d), as well as the size and shape of the sewer cross section. The *wetted area* (A) is the part of the sewer cross section occupied by the wastewater flow, while the *wetted perimeter* (P) is the part of the sewer perimeter in contact with the wastewater flow.⁵ The *surface width* (B) is the width of the flow surface.⁵ The geometric elements of Form 1 and Form 2 egg-shaped sewers are illustrated in Figure 4.

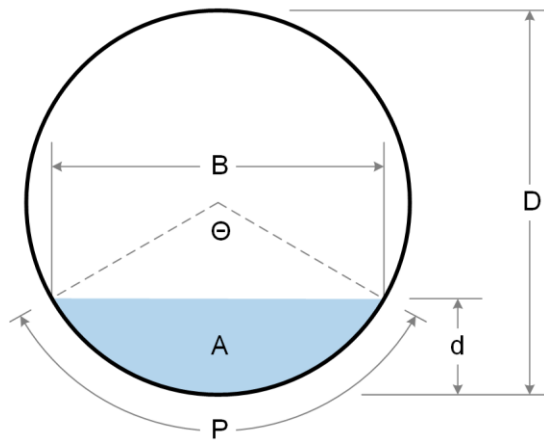
FIGURE 4: Geometric Elements of Various Partially Filled Egg-Shaped Sewers



Partially Filled Circular Sewer

Since an egg-shaped sewer is constructed from several circles, its geometric elements are based on the geometry of a partially filled circular sewer as illustrated in Figure 5 and defined in Equations (1) through (4).⁵

FIGURE 5: Geometric Elements of a Partially Filled Circle



$$\theta = 2 \cos^{-1} \left(1 - 2 \frac{d}{D} \right) \quad (1)$$

$$A = \frac{D^2}{8} (\theta - \sin \theta) \quad (2)$$

$$P = \frac{D}{2} \theta \quad (3)$$

$$B = D \sin \frac{\theta}{2} \quad (4)$$

where: θ = central angle, radians
 d = flow depth, m
 D = diameter, m
 A = wetted area, m²
 P = wetted perimeter, m
 B = surface width, m

Partially Filled Egg-Shaped Sewer – Form 1

Once the geometry of the Form 1 egg-shaped sewer is defined, several regions are identified with distinct geometric calculations. Three regions within the Form 1 egg-shaped sewer are detailed in Table 2.

TABLE 2: Geometric Regions of an Egg-Shaped Sewer – Form 1

Region	Depth
I	$0 \leq d \leq \frac{1}{10}D_T$
II	$\frac{1}{10}D_T < d \leq D_T$
III	$D_T < d \leq \frac{3}{2}D_T$

Geometric elements in Region I are based on Circle 1. Geometric elements in Region II are based on Circle 1 and Circle 2, and geometric elements in Region III are based on Circle 1, Circle 2, and Circle 3.

The wetted area of a partially filled Form 1 egg-shaped sewer is calculated using the compound equation shown in Equation (5).

$$A_{egg} = \quad (5)$$

$+A_1$ where $d = d$	$+A_2$ where $d = d$ $-A_2$ where $d = \frac{1}{10}D_T$ $-2D_T \left(d - \frac{1}{10}D_T \right)$ where $d = d$ $+A_1$ where $d = \frac{1}{10}D_T$	$+A_3$ where $d = d$ $-A_3$ where $d = D_T$ $+A_2$ where $d = D_T$ $-A_2$ where $d = \frac{1}{10}D_T$ $-\frac{9}{5}D_T^2$ $+A_1$ where $d = \frac{1}{10}D_T$
<i>when d is in Region I</i>	<i>when d is in Region II</i>	<i>when d is in Region III</i>
Circle 1	Circle 2	Circle 3
$\theta_1 = 2 \cos^{-1} \left(1 - 4 \frac{d}{D_T} \right)$ $A_1 = \frac{D_T^2}{32} (\theta_1 - \sin \theta_1)$	$\theta_2 = 2 \cos^{-1} \left[\frac{2}{3} \left(1 - \frac{d}{D_T} \right) \right]$ $A_2 = \frac{9}{8} D_T^2 (\theta_2 - \sin \theta_2)$	$\theta_3 = 2 \cos^{-1} \left[2 \left(1 - \frac{d}{D_T} \right) \right]$ $A_3 = \frac{D_T^2}{8} (\theta_3 - \sin \theta_3)$

Validation

Equations (5) through (7) are validated by computing the wetted area, wetted perimeter, and surface width at various flow depths for a Form 1 egg-shaped sewer with a specified transverse diameter (D_T) and comparing the results to reference sources and/or values derived from them. Results are provided in Table 4 for a Form 1 egg-shaped sewer where $D_T = 1.5$ m. The flow depths chosen for comparison include the maximum flow depth in Regions I, II, and III.

TABLE 4: Validation of Equations for a Partially Filled Form 1 Egg-Shaped Sewer when $D_T = 1.5$ m

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (5) (m ²)	Ref. (m ²)	Δ (%)	Eqn. (6) (m)	Ref. (m)	Δ (%)	Eqn. (7) (m)	Ref. (m)	Δ (%)
I	0.063	0.063	0%	0.695	0.695	0%	0.600	0.600	0%
II	1.701	1.701	0%	3.591	3.591	0%	1.500	1.500	0%
III	2.584	2.584	0%	5.947	5.947	0%	0.000	0.000	0%

Equations (8) through (10) are validated by computing the wetted area, wetted perimeter, and surface width at various flow depths for a Form 2 egg-shaped sewer with a specified transverse diameter (D_T) and comparing the results to reference sources and/or values derived from them. Results are provided in Table 5 for a Form 2 egg-shaped sewer where $D_T = 1.5$ m. The flow depths chosen for comparison include the maximum flow depth in Regions I, II, and III.

TABLE 5: Validation of Equations for a Partially Filled Form 2 Egg-Shaped Sewer when $D_T = 1.5$ m

Depth	Wetted Area			Wetted Perimeter			Surface Width		
	Eqn. (8) (m ²)	Ref. (m ²)	Δ (%)	Eqn. (9) (m)	Ref. (m)	Δ (%)	Eqn. (10) (m)	Ref. (m)	Δ (%)
I	0.009	0.009	0%	0.285	0.285	0%	0.259	0.259	0%
II	1.625	1.625	0%	3.525	3.525	0%	1.500	1.500	0%
III	2.509	2.509	0%	5.881	5.881	0%	0.000	0.000	0%

At the maximum flow depth in Region I, the wetted area, wetted perimeter, and surface width are equivalent to those of Circle 1 at the specified flow depth. The values obtained from Equations (5) through (7) for a Form 1 egg-shaped sewer and Equations (8) through (10) for a Form 2 egg-shaped sewer equal the expected values, as shown in Table 4 and Table 5, respectively.

At the maximum flow depth in Region II, no reference equations are available for comparison. However, the expected wetted area and wetted perimeter are derived by taking the full-pipe values from reference equations and subtracting the wetted area or wetted perimeter of Circle 3 when half-full. While no reference equation is available for the surface width, the geometry of Form 1 and Form 2 egg-shaped sewers dictates that it

is equal to the transverse diameter at this flow depth. The values obtained from Equations (5) through (7) for a Form 1 egg-shaped sewer and Equations (8) through (10) for a Form 2 egg-shaped sewer equal these derived values and geometric expectations, as shown in Table 4 and Table 5, respectively.

At the maximum flow depth in Region III, reference equations are available to calculate the wetted area and wetted perimeter under full-flow conditions.⁶ While no reference equation is available for the surface width, the geometry of Form 1 and Form 2 egg-shaped sewers dictates that it is zero at this flow depth. The values obtained from Equations (5) through (7) for a Form 1 egg-shaped sewer and Equations (8) through (10) for a Form 2 egg-shaped sewer equal those obtained using reference equations and geometric expectations, as shown in Table 4 and Table 5, respectively.

Hydraulic Elements

Once the geometric elements of a partially filled egg-shaped sewer are known, several *hydraulic elements* needed for commonly used wastewater engineering calculations can be determined. The *hydraulic radius* (R) is calculated using Equation (11) and is defined as the ratio of the wetted area to the wetted perimeter.⁵

$$R = \frac{A}{P} \quad (11)$$

where: R = hydraulic radius, m
 A = wetted area, m²
 P = wetted perimeter, m

The *hydraulic mean depth* (d_h) is calculated using Equation (12) and is defined as the ratio of the wetted area to the surface width.⁵

$$d_h = \frac{A}{B} \quad (12)$$

where: d_h = hydraulic mean depth, m
 A = wetted area, m²
 B = surface width, m

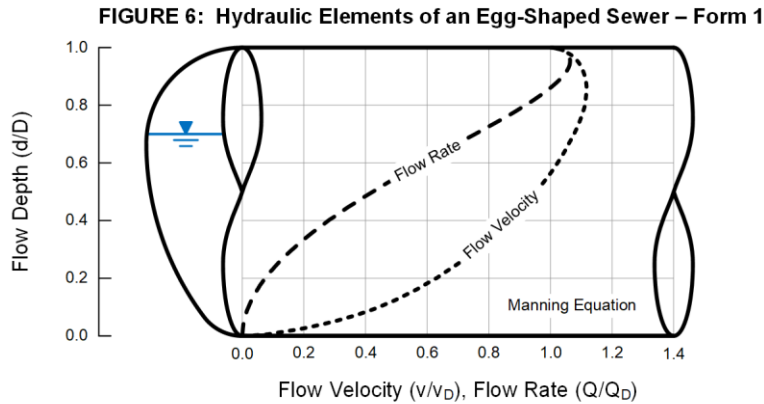
The hydraulic radius is used in the Manning Equation as shown in Equations (13) and (14) to estimate the flow velocity (v) and flow rate (Q) under uniform flow conditions.¹

$$v = \frac{1}{n} R^{2/3} S^{1/2} \quad (13)$$

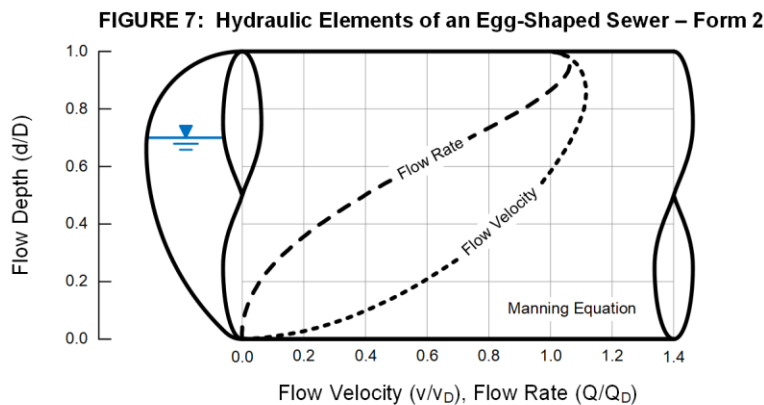
$$Q = \frac{1}{n} A R^{2/3} S^{1/2} \quad (14)$$

where: v = flow velocity, m/s
 n = roughness coefficient
 R = hydraulic radius, m
 S = slope of the energy gradient
 Q = flow rate, m³/s
 A = wetted area, m²

Hydraulic elements curves prepared using the Manning Equation for a Form 1 egg-shaped sewer at various relative flow depths are shown in Figure 6.



Hydraulic elements curves prepared using the Manning Equation for a Form 2 egg-shaped sewer at various relative flow depths are shown in Figure 7.



Other important properties enabled once the geometric elements of a partially full egg-shaped sewer are known include the Froude Number, Reynolds Number, and shear stress.^{5,7}

Conclusion

Combined sewers are designed and constructed with a wide variety of sewer cross sections. While the circular section is most common, other non-circular sections have also been used over the years. Working with combined sewers requires an understanding of the geometric and hydraulic elements of these non-circular sections when encountered.

The geometric elements of the Form 1 and Form 2 egg-shaped sewers flowing partially full have been defined, and equations for the wetted area, wetted perimeter, and surface width are provided for each. The result is a concise reference for wastewater professionals working with egg-shaped sewers in combined sewer systems.

Symbols and Notation

The following symbols and notation are used in this paper:

VARIABLES

θ	= central angle, radians
d	= flow depth, m
D	= diameter, m
A	= wetted area, m ²
P	= wetted perimeter, m
B	= surface width, m
R	= hydraulic radius, m
v	= flow velocity, m/s
Q	= flow rate, m ³ /s
n	= roughness coefficient
S	= slope of the energy gradient

SUBSCRIPTS

T	= transverse
V	= vertical
S	= spring line
1	= Circle 1
2	= Circle 2
3	= Circle 3
h	= hydraulic

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