Scattergraph Principles and Practice

Camp's Varying Roughness Coefficient Applied to the Manning Equation

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ABSTRACT

The Manning Equation is an empirical formula commonly used to design sewer systems. Most design methods assume that the roughness coefficient is constant, but historical research has shown that it varies as a function of flow depth. The use of a constant or varying roughness coefficient is often left to the discretion of the design engineer.

The same consideration applies to scattergraph methods that correlate the Manning Equation to flow monitor data. The Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method have been previously reported using a constant roughness coefficient. A modification is presented in this paper to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%.

Examples of these methods using a constant and varying roughness coefficient are provided from flow monitor locations throughout the United States. Laboratory research by the authors is also provided and indicates that the use of a varying roughness coefficient provides a more accurate determination of sewer capacity.

KEY WORDS Flow Monitoring, Manning Equation, Scattergraph, Roughness Coefficient

Introduction

The scattergraph is a graphical tool that provides insight into sewer performance through a simple and intuitive display of flow monitor data. The resulting patterns form characteristic signatures that reveal important information about conditions within a sewer and the impact that these conditions have on sewer capacity.[1] The Manning Equation is an important component of the scattergraph and can be applied using a variety of methods. The Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method have been previously reported using a constant roughness coefficient.[2] A modification is presented to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%.

Manning Equation

The Manning Equation is an empirical formula used to design sewer systems. The most common expression of this formula is provided in Equation (1).

$$v = \frac{1.486}{n} R^{2/3} S^{1/2} \quad (1)$$

where:
- v = flow velocity, ft/s
- n = roughness coefficient
- R = hydraulic radius, ft
- S = slope of the energy gradient

Several assumptions are generally made with respect to the Manning Equation: the roughness coefficient is constant, and the slope of the energy gradient equals the slope of the pipe.[3] However, historical research reported by Camp and others has shown that the roughness coefficient varies as a function of flow depth.[4] This variation can be expressed in general terms as provided in Equation (2).

$$n = n_D f(d) \quad (2)$$

where:
- n = roughness coefficient
- n_D = roughness coefficient at d = D
- d = flow depth, ft
- D = diameter, ft

The varying roughness coefficient is incorporated into the Manning Equation by direct substitution as shown in Equation (3).

$$v = \frac{1.486}{n_D f(d)} R^{2/3} S^{1/2} \quad (3)$$

where:
- v = flow velocity, ft/s
- n_D = roughness coefficient at d = D
- d = flow depth, ft
- D = diameter, ft
- R = hydraulic radius, ft
- S = slope of the energy gradient

Based on this revised assumption, the Manning Equation can be algebraically rearranged such that the constant parameters are consolidated into a single coefficient, defined as the *hydraulic coefficient*, and restated as provided in Equation (4). This expression is useful in subsequent discussions.

$$v = 1.486 \frac{C}{f(d)} R^{2/3} \qquad (4)$$

where: v = flow velocity, ft/s
d = flow depth, ft
C = hydraulic coefficient
R = hydraulic radius, ft

The varying roughness coefficient has been historically reported in a graphical format. However, this relationship can also be described in equation form. A fourth order polynomial approximation of Camp's varying roughness coefficient is provided in Equation (5):

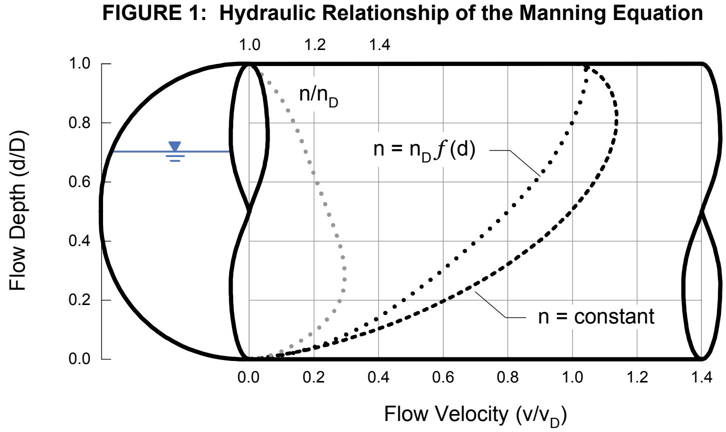
$$f(d) = 1.04 + 2.30(d/D) - 6.86(d/D)^2 + 7.79(d/D)^3 - 3.27(d/D)^4 \qquad (5)$$

where: d = flow depth, ft
D = diameter, ft

Other equations have also been reported in the literature by various researchers, including Zaghloul, Wong and Zhou, and Akgiray.[5, 6, 7, 8] These equations are mathematically interchangeable with Equation (5) in subsequent discussions.

The relationship between flow depth and velocity described by the Manning Equation is depicted in Figure 1 as a *pipe curve* and provides a convenient reference to evaluate flow monitor data. The relationships described using a constant roughness coefficient (---) and a varying roughness coefficient (···) are provided for comparison.

FIGURE 1: Hydraulic Relationship of the Manning Equation

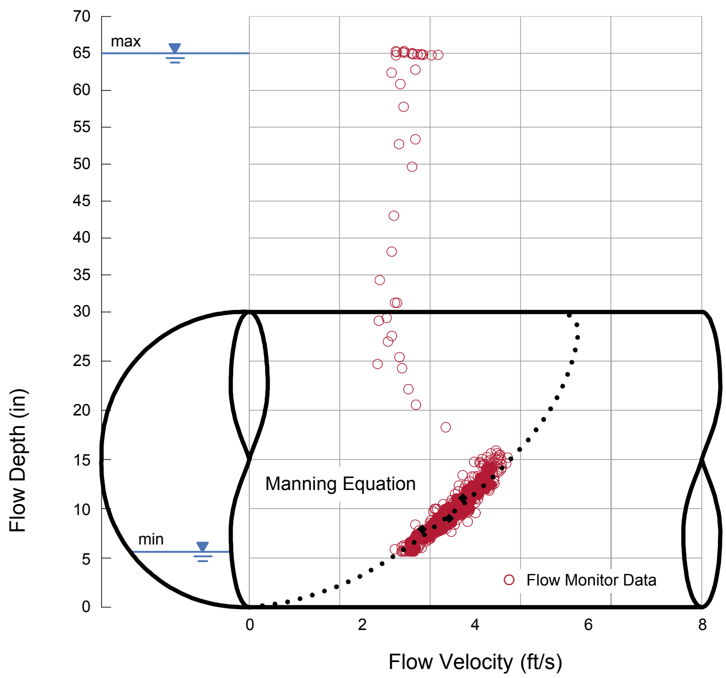


Manning Methods

The Manning Equation is also used to describe the performance of existing sewers by evaluating flow monitor data on a scattergraph, as shown in Figure 2. The Manning Equation is used to generate a pipe curve which is then compared to actual flow monitor data (○). This data may agree or disagree with the Manning Equation, depending on actual conditions at the monitoring location. In either case, important information can be learned about the performance of a sewer and its effect on sewer capacity.[9]

For example, the flow monitor data shown in Figure 2 indicate that this sewer operates as expected up to a flow depth of about 15 inches. However, as backwater conditions develop, flow conditions become deeper and slower and are revealed on the scattergraph as a departure from the pipe curve, resulting in surcharge and overflow conditions at a much lower capacity than expected.[10] Three manual confirmations (◆) are also shown and provide a means to evaluate the accuracy of the flow monitor.

FIGURE 2: Scattergraph of Flow Depth and Velocity Data

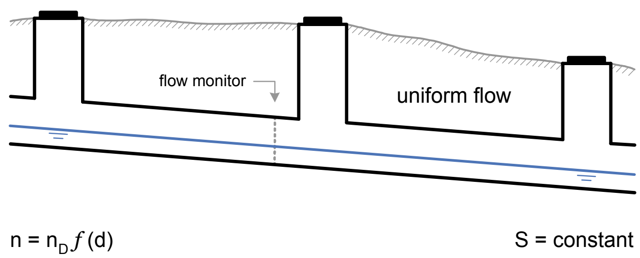


The Manning Equation is an important component of the scattergraph and can be applied using three different methods, defined as the Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method. The Design Method uses the Manning Equation to describe a relationship between flow depth and velocity using a specified roughness coefficient and pipe slope. This relationship is then compared with actual flow monitor data. The Lanfear-Coll Method and the Stevens-Schutzbach Method use curve fitting techniques to correlate the Manning Equation directly to such data. These methods have been previously reported using a constant roughness coefficient.[2] Modifications are presented in the following sections to incorporate a varying roughness coefficient into these methods.

Design Method

The *Design Method* uses the Manning Equation with a specified roughness coefficient and pipe slope and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 3.

FIGURE 3: General Assumptions of the Design Method



The Design Method incorporates the Manning Equation as expressed in Equation (6) and the hydraulic radius as defined in Equation (7).

$$v = 1.486 \frac{C_{DM}}{f(d)} R_{DM}^{2/3} \tag{6}$$

$$R_{DM} = \frac{A}{P} \tag{7}$$

where:
- v = flow velocity, ft/s
- C_{DM} = hydraulic coefficient
- d = flow depth, ft
- R_{DM} = hydraulic radius, ft
- A = wetted cross-section, ft^2
- P = wetted perimeter, ft

The roughness coefficient and the pipe slope are specified based on design assumptions, as-built documentation, or field observations and are used to calculate the hydraulic coefficient as shown in Equation (8).

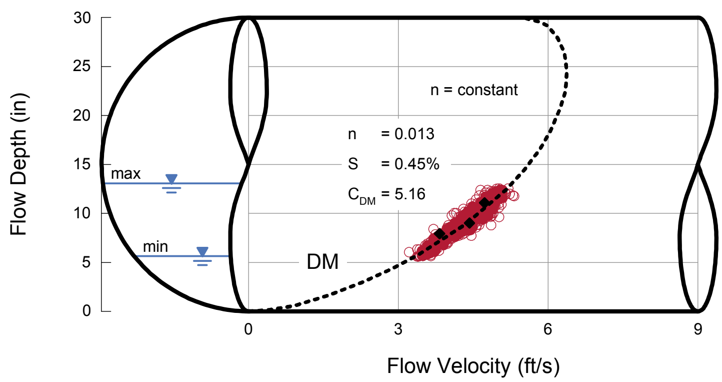
$$C_{DM} = \frac{1}{n_D} S^{1/2} \tag{8}$$

where:
- C_{DM} = hydraulic coefficient
- n = roughness coefficient
- S = pipe slope

If the value of the roughness coefficient is known at a given depth, n_D is calculated using Equation (2) and Equation (5). The Design Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 \leq d \leq D$. The application of the Design Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE

Flow monitor data are obtained from a 30-in sewer, as shown in the scattergraph below. A pipe curve has been previously constructed using the Design Method with a constant roughness coefficient and the roughness coefficient (n) and pipe slope (S) provided below.[2]



Use the Design Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare this result to the full-pipe capacity determined with a constant roughness coefficient.

EXAMPLE

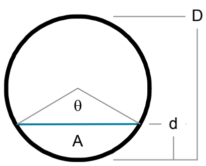
Solution: Calculate the hydraulic coefficient, construct pipe curve, and estimate sewer capacity

(a) Calculate n_D assuming n = 0.013 at d = 9 inches n_D = 0.010

(b) Calculate C_{DM} assuming n_D = 0.010 and S = 0.45% C_{DM} = 6.71

(c) Calculate v_{DM} for $0 \leq d \leq D$

For a circular sewer,[11]



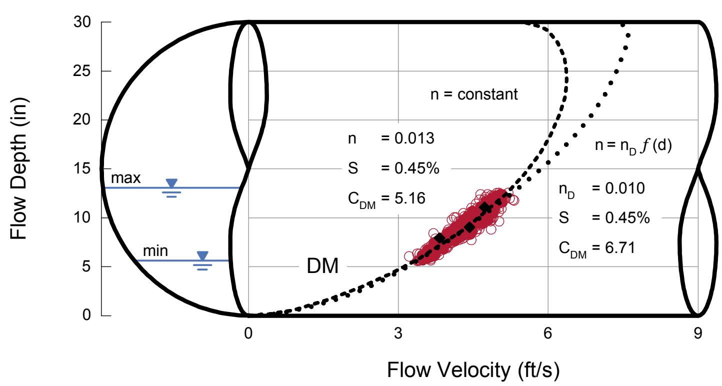
$\theta = 2\cos^{-1}(1 - 2d/D)$

$A = (D^2/8)(\theta - \sin\theta)$

$P = D\theta/2$

d	θ	A	P	R_{DM}	f(d)	$R_{DM}^{2/3}/f$(d)	v_{DM}
in	°	ft²	ft	ft		ft$^{2/3}$	ft/s
0	0	0.00	0.00	—	1.04	—	—
5	96	0.54	2.10	0.26	1.27	0.32	3.17
10	141	1.43	3.08	0.47	1.29	0.46	4.63
15	180	2.45	3.93	0.63	1.24	0.59	5.86
20	219	3.48	4.78	0.73	1.19	0.68	6.80
25	264	4.37	5.75	0.76	1.12	0.74	7.39
30	360	4.91	7.85	0.63	1.00	0.73	7.29

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation using the Design Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

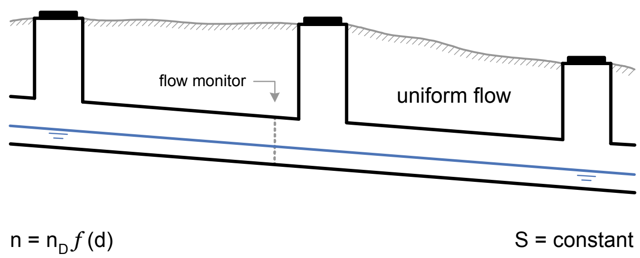
(d) Calculate Q_{DM} for d = D

The full-pipe capacity is calculated using the Continuity Equation, $Q_{DM} = Av_{DM}$, where Q_{DM} = 4.91 ft² x 5.61 ft/s = 27.5 ft³/s or 17.8 MGD, about 30% greater than the corresponding values determined using a constant roughness coefficient.[2]

Lanfear-Coll Method

The Lanfear-Coll Method uses a curve fitting technique to fit the Manning Equation to flow monitor data and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 4.

FIGURE 4: General Assumptions of the Lanfear-Coll Method



This method is applicable to flow monitor data obtained under uniform flow conditions and incorporates the Manning Equation as expressed in Equation (9) and the hydraulic radius as defined in Equation (10).

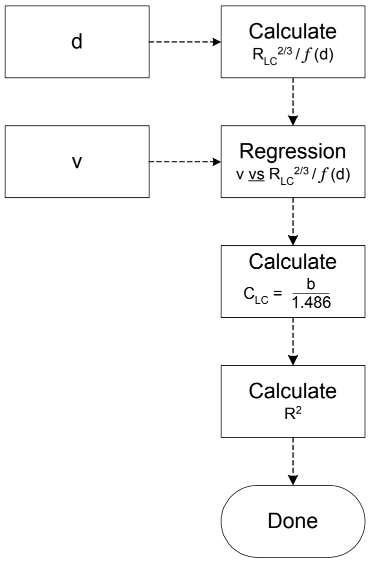
$$v = 1.486 \frac{C_{LC}}{f(d)} R_{LC}^{2/3} \tag{9}$$

$$R_{LC} = \frac{A}{P} \tag{10}$$

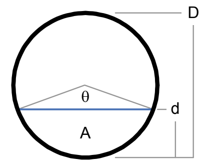
where:
- v = flow velocity, ft/s
- C_{LC} = hydraulic coefficient
- d = flow depth, ft
- R_{LC} = hydraulic radius, ft
- A = wetted cross-section, ft^2
- P = wetted perimeter, ft

This method provides an implicit solution to the Manning Equation and requires no direct knowledge of the roughness coefficient or the slope of the energy gradient. Flow depth and velocity data are used to calculate the hydraulic coefficient based on a least-squares regression of Equation (9) using a varying roughness coefficient, as described in Figure 5. Regression results are characterized using the coefficient of determination.[12]

FIGURE 5: Regression Using the Lanfear-Coll Method



For a circular sewer,



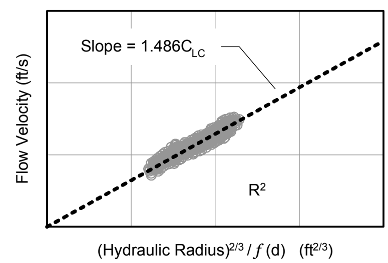
$$\theta = 2\cos^{-1}(1 - 2d/D)$$
$$A = (D^2/8)(\theta - \sin\theta)$$
$$P = D\theta/2$$

$$R_{LC} = \frac{A}{P}$$

Restate Equation (9) as y = a + bx using direct substitution, where:

$$x = R_{LC}^{2/3} / f(d)$$
$$y = v_{LC}$$
$$a = 0$$
$$b = 1.486C_{LC}$$

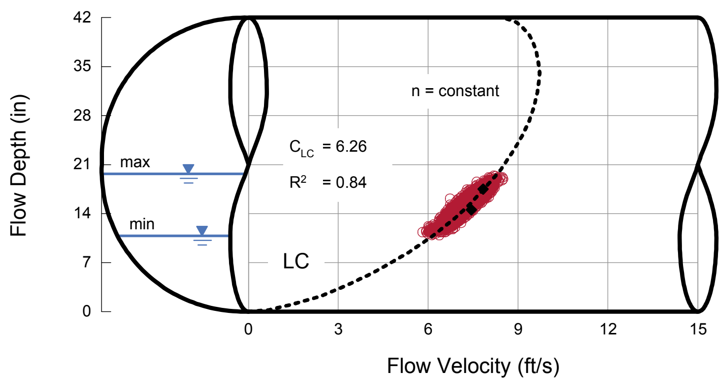
Perform least squares regression.



The Lanfear-Coll Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 \le d \le D$. The application of the Lanfear-Coll Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE

Flow monitor data are obtained from a 42-in sewer, as shown in the scattergraph below. Tabular data are provided on the following page. A pipe curve has been previously constructed using the Lanfear-Coll Method with a constant roughness coefficient.[2]



Use the Lanfear-Coll Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare this result to the full-pipe capacity determined with a constant roughness coefficient.

EXAMPLE

Solution: Calculate the hydraulic coefficient

(a) Calculate $R_{LC}^{2/3} / f(d)$

date	time	d	v	θ	A	P	R_{LC}	$f(d)$	$R_{LC}^{2/3}/f(d)$
mm/dd	hh:mm	in	ft/s	°	ft^2	ft	ft		$ft^{2/3}$
11/01	00:00	13.99	7.18	141	2.80	4.31	0.65	1.29	0.58
11/01	00:15	14.03	7.40	141	2.82	4.31	0.65	1.29	0.58
11/01	00:30	13.71	7.11	139	2.73	4.26	0.64	1.29	0.57
11/01	00:45	13.59	7.15	139	2.70	4.24	0.64	1.29	0.57
11/01	01:00	13.22	6.89	137	2.59	4.17	0.62	1.30	0.56
11/01	01:15	13.16	7.00	136	2.58	4.16	0.62	1.30	0.56
11/01	01:30	13.14	6.82	136	2.57	4.16	0.62	1.30	0.56
11/01	01:45	13.01	6.71	135	2.54	4.13	0.61	1.30	0.56
11/01	02:00	12.81	6.71	134	2.48	4.10	0.61	1.30	0.55
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
11/30	23:45	15.64	7.22	150	3.26	4.59	0.71	1.28	0.62
			v_{avg} ← 7.17						

(b) Calculate C_{LC} and R^2 based on a least squares regression

date	time	x	y	xy	x^2	v_{LC}	$(v_{LC} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$ft^{2/3}$	ft/s	$ft^{5/3}/s$	$ft^{4/3}$	ft/s	$(ft/s)^2$	$(ft/s)^2$
11/01	00:00	0.58	7.18	4.17	0.34	6.95	0.053	0.000
11/01	00:15	0.58	7.40	4.31	0.34	6.96	0.192	0.052
11/01	00:30	0.57	7.11	4.09	0.33	6.87	0.058	0.004
11/01	00:45	0.57	7.15	4.09	0.33	6.84	0.099	0.000
11/01	01:00	0.56	6.89	3.88	0.32	6.73	0.026	0.079
11/01	01:15	0.56	7.00	3.93	0.32	6.71	0.083	0.029
11/01	01:30	0.56	6.82	3.82	0.31	6.71	0.013	0.124
11/01	01:45	0.56	6.71	3.74	0.31	6.67	0.002	0.213
11/01	02:00	0.55	6.71	3.71	0.31	6.61	0.010	0.213
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
11/30	23:45	0.62	7.22	4.48	0.38	7.41	0.037	0.002
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,880 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{LC} = \frac{\sum xy \, / \sum x^2}{1.486}$$

$$R^2 = 1 - \frac{SSE}{SYY}$$

Based on the regression results, C_{LC} = 8.05 and R^2 = 0.82.

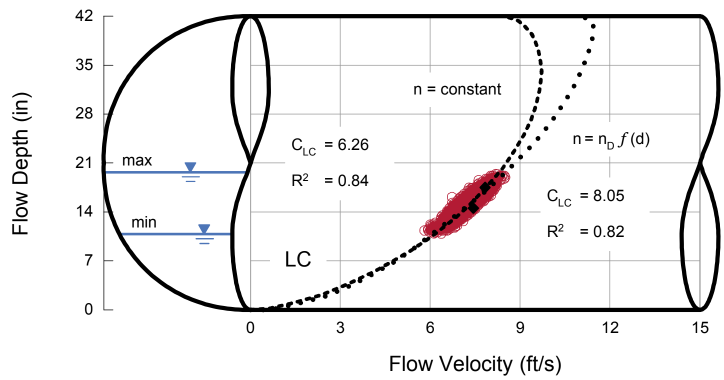
EXAMPLE

Solution: Construct pipe curve and estimate sewer capacity

(c) Calculate v_{LC} for $0 \le d \le D$

d	θ	A	P	R_{LC}	$f(d)$	$R_{LC}^{2/3}/f(d)$	v_{LC}
in	°	ft^2	ft	ft		$ft^{2/3}$	ft/s
0	0	0.00	0.00	—	1.04	—	—
7	96	1.05	2.94	0.36	1.27	0.40	4.76
14	141	2.81	4.31	0.65	1.29	0.58	6.95
21	180	4.81	5.50	0.88	1.24	0.74	8.79
28	219	6.81	6.69	1.02	1.19	0.85	10.20
35	264	8.57	8.05	1.06	1.12	0.93	11.09
42	360	9.62	11.00	0.88	1.00	0.91	10.94

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation fitted to observed flow depth and velocity data using the Lanfear-Coll Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

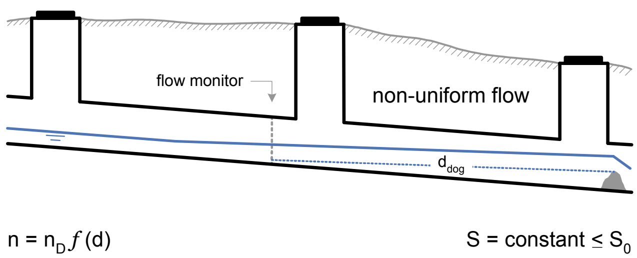
(d) Calculate Q_{LC} for d = D

The full-pipe capacity is calculated using the Continuity Equation, $Q_{LC} = Av_{LC}$, where Q_{LC} = 9.62 ft^2 x 10.94 ft/s = 105.2 ft^3/s or 68.0 MGD, about 29% greater than the corresponding values determined using a constant roughness coefficient.[2]

Stevens-Schutzbach Method

The Stevens-Schutzbach Method uses an iterative curve fitting technique to fit the Manning Equation to flow monitor data and has been previously reported using a constant roughness coefficient.[2] A modification is described here to incorporate a varying roughness coefficient into this method. The Manning Equation is applied using this modification under the general assumptions shown in Figure 6.

FIGURE 6: General Assumptions of the Stevens-Schutzbach Method



This method is applicable to flow monitor data obtained under uniform flow conditions or non-uniform flow conditions resulting from a variety of downstream obstructions, or *dead dogs*. Examples include offset joints, debris, and other related conditions. The modified Stevens-Schutzbach Method incorporates the Manning Equation as expressed in Equation (11) and the hydraulic radius as defined in Equation (12).

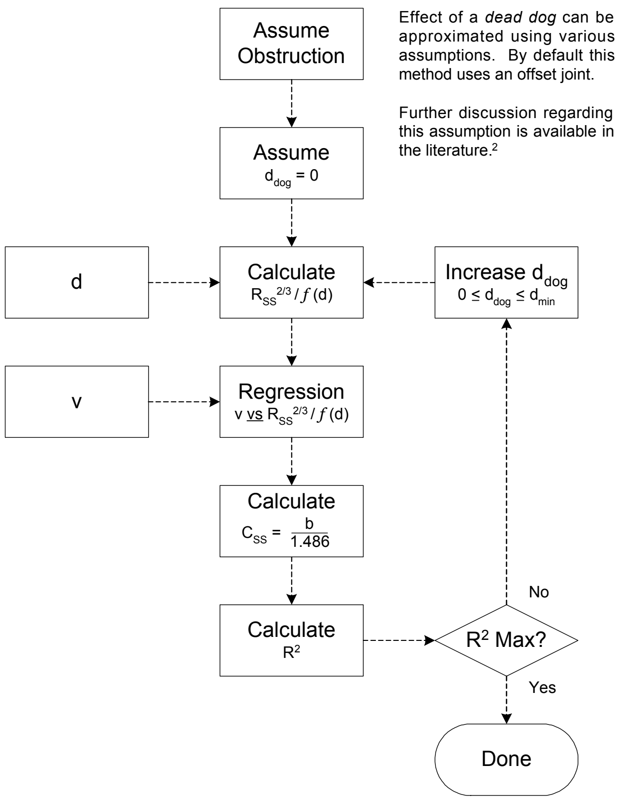
$$v = 1.486 \frac{C_{SS}}{f(d)} R_{SS}^{2/3} \tag{11}$$

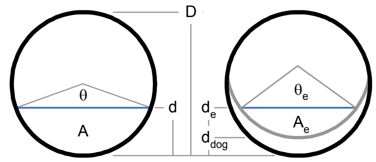
$$R_{SS} = \frac{A_e}{P} \tag{12}$$

where:
- v = flow velocity, ft/s
- C_{SS} = hydraulic coefficient
- d = flow depth, ft
- R_{SS} = hydraulic radius, ft
- A_e = effective wetted cross-section, ft^2
- P = wetted perimeter, ft

Note that the definition of the hydraulic radius is modified from the traditional definition and requires certain assumptions regarding the shape and magnitude of the *dead dog*. Based on these assumptions, flow depth and velocity data are used to calculate the hydraulic coefficient based on an iterative least-squares regression method using a varying roughness coefficient, as described in Figure 7. The magnitude of the *dead dog* (d_{dog}) is varied in successive iterations until the coefficient of determination is maximized.

FIGURE 7: Regression Using the Stevens-Schutzbach Method





For a circular sewer,

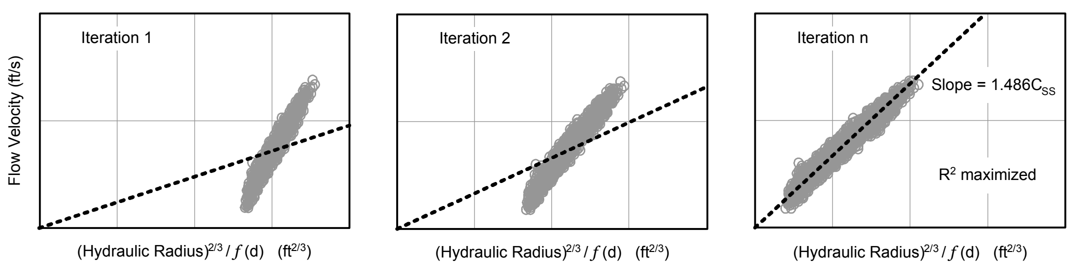
$$\theta = 2\cos^{-1}(1 - 2d/D)$$
$$A = (D^2/8)(\theta - \sin\theta)$$
$$P = D\theta/2$$

$$d_e = d - d_{dog}$$
$$\theta_e = 2\cos^{-1}(1 - 2d_e/D)$$
$$A_e = (D^2/8)(\theta_e - \sin\theta_e)$$
$$R_{SS} = \frac{A_e}{P}$$

Restate Equation (8) as $y = a + bx$ using direct substitution, where:

$$x = R_{SS}^{2/3} / f(d)$$
$$y = v_{SS}$$
$$a = 0$$
$$b = 1.486C_{SS}$$

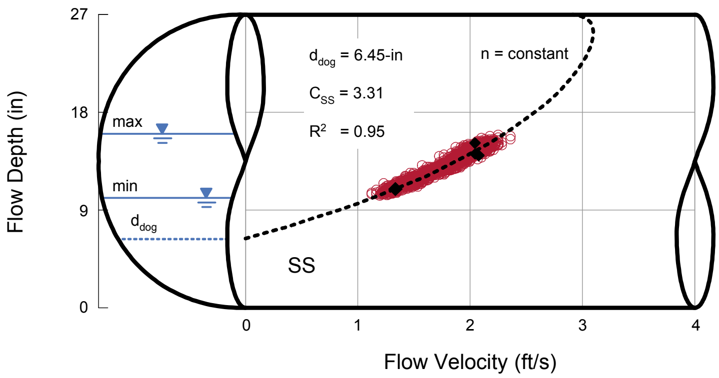
Perform iterative least squares regression.



The Stevens-Schutzbach Method is then used to generate a pipe curve which is compared to actual flow monitor data on a scattergraph. If the data agree with the pipe curve, then this method can be used to estimate the full-pipe capacity of the sewer, assuming the assumptions of this method remain valid at the monitoring location from $0 \le d \le D$. The application of the Stevens-Schutzbach Method using a varying roughness coefficient is demonstrated in the following example.

EXAMPLE

Flow monitor data are obtained from a 27-in sewer, as shown in the scattergraph below. Tabular data are provided on the following page. A pipe curve has been previously constructed using the Stevens-Schutzbach Method with a constant roughness coefficient.[2]



Use the Stevens-Schutzbach Method with a varying roughness coefficient to construct a pipe curve on the scattergraph and estimate the full-pipe capacity of this sewer. Compare this result to the full-pipe capacity determined with a constant roughness coefficient.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 1

(a) Assume d_{dog} = 0.00 in. Calculate $R_{SS}^{2/3}/f(d)$

date	time	d	v	d_e	θ_e	A_e	θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	in	ft/s	in	°	ft^2	°	ft	ft		$ft^{2/3}$
08/01	00:00	14.34	2.11	14.34	187	2.15	187	3.67	0.58	1.23	0.57
08/01	00:15	14.08	2.03	14.08	185	2.10	185	3.63	0.58	1.24	0.56
08/01	00:30	13.91	1.99	13.91	183	2.06	183	3.60	0.57	1.24	0.56
08/01	00:45	13.81	1.96	13.81	183	2.05	183	3.59	0.57	1.24	0.55
08/01	01:00	13.48	1.99	13.48	180	1.98	180	3.53	0.56	1.24	0.55
08/01	01:15	13.14	1.92	13.14	177	1.92	177	3.47	0.55	1.25	0.54
08/01	01:30	12.93	1.84	12.93	175	1.88	175	3.44	0.55	1.25	0.53
08/01	01:45	13.04	1.88	13.04	176	1.90	176	3.46	0.55	1.25	0.54
08/01	02:00	12.88	1.80	12.88	175	1.87	175	3.43	0.55	1.25	0.53
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	14.19	2.07	14.19	186	2.12	186	3.65	0.58	1.24	0.56
	10.35 →	d_{min}	v_{avg}	← 1.86							

(b) Calculate C_{SS} and R^2 based on a least squares regression

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$ft^{2/3}$	ft/s	$ft^{5/3}/s$	$ft^{4/3}$	ft/s	$(ft/s)^2$	$(ft/s)^2$
08/01	00:00	0.57	2.11	1.20	0.32	1.93	0.031	0.063
08/01	00:15	0.56	2.03	1.14	0.31	1.91	0.014	0.030
08/01	00:30	0.56	1.99	1.11	0.31	1.90	0.008	0.017
08/01	00:45	0.55	1.96	1.09	0.31	1.89	0.004	0.010
08/01	01:00	0.55	1.99	1.09	0.30	1.87	0.015	0.017
08/01	01:15	0.54	1.92	1.04	0.29	1.84	0.006	0.004
08/01	01:30	0.53	1.84	0.98	0.29	1.82	0.000	0.000
08/01	01:45	0.54	1.88	1.01	0.29	1.83	0.002	0.000
08/01	02:00	0.53	1.80	0.96	0.28	1.82	0.000	0.003
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.56	2.07	1.17	0.32	1.92	0.022	0.045
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \frac{\sum xy \,/\, \sum x^2}{1.486} \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.30 and R^2 = 0.61. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 2

(a) Assume d_{dog} = 1.00 in. Calculate $R_{SS}^{2/3}/f(d)$

date	time	d	v	d_e	θ_e	A_e	θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	in	ft/s	in	°	ft²	°	ft	ft		ft^(2/3)
08/01	00:00	14.34	2.11	13.34	179	1.96	187	3.67	0.53	1.23	0.53
08/01	00:15	14.08	2.03	13.08	176	1.91	185	3.63	0.53	1.24	0.53
08/01	00:30	13.91	1.99	12.91	175	1.88	183	3.60	0.52	1.24	0.52
08/01	00:45	13.81	1.96	12.81	174	1.86	183	3.59	0.52	1.24	0.52
08/01	01:00	13.48	1.99	12.48	171	1.80	180	3.53	0.51	1.24	0.51
08/01	01:15	13.14	1.92	12.14	168	1.73	177	3.47	0.50	1.25	0.50
08/01	01:30	12.93	1.84	11.93	167	1.69	175	3.44	0.49	1.25	0.50
08/01	01:45	13.04	1.88	12.04	168	1.71	176	3.46	0.50	1.25	0.50
08/01	02:00	12.88	1.80	11.88	166	1.69	175	3.43	0.49	1.25	0.50
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	14.19	2.07	13.19	177	1.93	186	3.65	0.53	1.24	0.53

10.35 → d_{min} v_{avg} ← 1.86

(b) Calculate C_{SS} and R^2 based on a least squares regression

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	ft^(2/3)	ft/s	ft^(5/3)/s	ft^(4/3)	ft/s	(ft/s)²	(ft/s)²
08/01	00:00	0.53	2.11	1.12	0.28	1.94	0.028	0.063
08/01	00:15	0.53	2.03	1.07	0.28	1.92	0.012	0.030
08/01	00:30	0.52	1.99	1.04	0.27	1.91	0.007	0.017
08/01	00:45	0.52	1.96	1.02	0.27	1.90	0.004	0.010
08/01	01:00	0.51	1.99	1.02	0.26	1.87	0.015	0.017
08/01	01:15	0.50	1.92	0.97	0.25	1.84	0.007	0.004
08/01	01:30	0.50	1.84	0.92	0.25	1.82	0.001	0.000
08/01	01:45	0.50	1.88	0.94	0.25	1.83	0.003	0.000
08/01	02:00	0.50	1.80	0.89	0.25	1.81	0.000	0.003
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.53	2.07	1.10	0.28	1.93	0.019	0.045
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \frac{\sum xy \; / \; \sum x^2}{1.486} \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.45 and R^2 = 0.68. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration 3

(a) Assume d_{dog} = 2.00 in. Calculate $R_{SS}^{2/3}/f(d)$

date	time	d	v	d_e	θ_e	A_e	θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	in	ft/s	in	o	ft^2	o	ft	ft		$ft^{2/3}$
08/01	00:00	14.34	2.11	12.34	170	1.77	187	3.67	0.48	1.23	0.50
08/01	00:15	14.08	2.03	12.08	168	1.72	185	3.63	0.47	1.24	0.49
08/01	00:30	13.91	1.99	11.91	166	1.69	183	3.60	0.47	1.24	0.49
08/01	00:45	13.81	1.96	11.81	166	1.67	183	3.59	0.47	1.24	0.48
08/01	01:00	13.48	1.99	11.48	163	1.61	180	3.53	0.46	1.24	0.48
08/01	01:15	13.14	1.92	11.14	160	1.55	177	3.47	0.45	1.25	0.47
08/01	01:30	12.93	1.84	10.93	158	1.51	175	3.44	0.44	1.25	0.46
08/01	01:45	13.04	1.88	11.04	159	1.53	176	3.46	0.44	1.25	0.46
08/01	02:00	12.88	1.80	10.88	158	1.50	175	3.43	0.44	1.25	0.46
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	14.19	2.07	12.19	169	1.74	186	3.65	0.48	1.24	0.49
	10.35 →	d_{min}	v_{avg}	← 1.86							

(b) Calculate C_{SS} and R^2 based on a least squares regression

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$ft^{2/3}$	ft/s	$ft^{5/3}/s$	$ft^{4/3}$	ft/s	$(ft/s)^2$	$(ft/s)^2$
08/01	00:00	0.50	2.11	1.05	0.25	1.96	0.024	0.063
08/01	00:15	0.49	2.03	1.00	0.24	1.93	0.010	0.030
08/01	00:30	0.49	1.99	0.97	0.24	1.91	0.006	0.017
08/01	00:45	0.48	1.96	0.95	0.24	1.90	0.003	0.010
08/01	01:00	0.48	1.99	0.95	0.23	1.87	0.015	0.017
08/01	01:15	0.47	1.92	0.90	0.22	1.83	0.008	0.004
08/01	01:30	0.46	1.84	0.85	0.21	1.81	0.001	0.000
08/01	01:45	0.46	1.88	0.87	0.22	1.82	0.003	0.000
08/01	02:00	0.46	1.80	0.83	0.21	1.80	0.000	0.003
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.49	2.07	1.02	0.24	1.94	0.017	0.045
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \frac{\sum xy \,/\, \sum x^2}{1.486} \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 2.64 and R^2 = 0.74. R^2 is not maximized.

EXAMPLE

Solution: Calculate the hydraulic coefficient - Iteration n

(a) Assume d_{dog} = 6.01 in. Calculate $R_{SS}^{2/3}/f(d)$

date	time	d	v	d_e	θ_e	A_e	θ	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$
mm/dd	hh:mm	in	ft/s	in	°	ft^2	°	ft	ft		$ft^{2/3}$
08/01	00:00	14.34	2.11	8.33	135	1.04	187	3.67	0.28	1.23	0.35
08/01	00:15	14.08	2.03	8.07	133	1.00	185	3.63	0.28	1.24	0.34
08/01	00:30	13.91	1.99	7.90	131	0.97	183	3.60	0.27	1.24	0.34
08/01	00:45	13.81	1.96	7.80	130	0.95	183	3.59	0.27	1.24	0.33
08/01	01:00	13.48	1.99	7.47	127	0.90	180	3.53	0.25	1.24	0.32
08/01	01:15	13.14	1.92	7.13	124	0.84	177	3.47	0.24	1.25	0.31
08/01	01:30	12.93	1.84	6.92	122	0.81	175	3.44	0.23	1.25	0.30
08/01	01:45	13.04	1.88	7.03	123	0.82	176	3.46	0.24	1.25	0.31
08/01	02:00	12.88	1.80	6.87	121	0.80	175	3.43	0.23	1.25	0.30
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	14.19	2.07	8.18	134	1.02	186	3.65	0.28	1.24	0.35
	10.35 →	d_{min}	v_{avg}	← 1.86							

(b) Calculate C_{SS} and R^2 based on a least squares regression

date	time	x	y	xy	x^2	v_{SS}	$(v_{SS} - v)^2$	$(v - v_{avg})^2$
mm/dd	hh:mm	$ft^{2/3}$	ft/s	$ft^{5/3}/s$	$ft^{4/3}$	ft/s	$(ft/s)^2$	$(ft/s)^2$
08/01	00:00	0.35	2.11	0.74	0.12	2.02	0.008	0.063
08/01	00:15	0.34	2.03	0.69	0.12	1.97	0.003	0.030
08/01	00:30	0.34	1.99	0.67	0.11	1.94	0.002	0.017
08/01	00:45	0.33	1.96	0.65	0.11	1.92	0.001	0.010
08/01	01:00	0.32	1.99	0.64	0.10	1.86	0.017	0.017
08/01	01:15	0.31	1.92	0.60	0.10	1.79	0.016	0.004
08/01	01:30	0.30	1.84	0.56	0.09	1.75	0.008	0.000
08/01	01:45	0.31	1.88	0.58	0.09	1.77	0.011	0.000
08/01	02:00	0.30	1.80	0.54	0.09	1.74	0.003	0.003
. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .	. . .
08/21	23:45	0.35	2.07	0.72	0.12	1.99	0.006	0.045
				$\sum xy$	$\sum x^2$		SSE	SYY

For this example, a total of 2,016 data points were used. Complete calculations are available in a spreadsheet that accompanies this technical paper.

$$C_{SS} = \frac{\sum xy \,/\, \sum x^2}{1.486} \qquad R^2 = 1 - \frac{SSE}{SYY}$$

Based on this iteration, C_{SS} = 3.88 and R^2 = 0.95. R^2 is maximized.

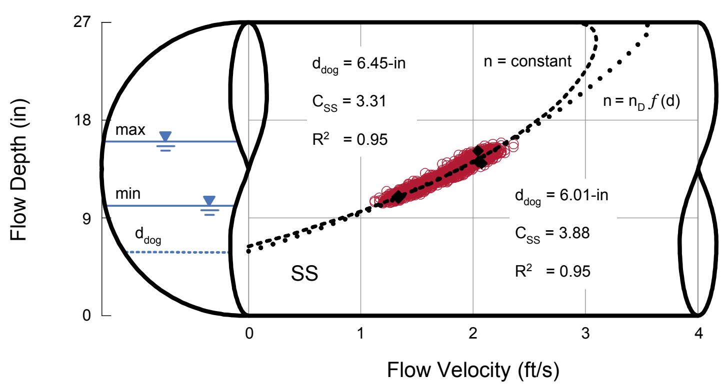
EXAMPLE

Solution: Construct pipe curve and estimate sewer capacity

(c) Calculate v_{SS} for $0 \leq d \leq D$

d	d_e	θ_e	A_e	θ	A	P	R_{SS}	$f(d)$	$R_{SS}^{2/3}/f(d)$	v_{SS}
in	in	°	ft^2	°	ft^2	ft	ft		$ft^{2/3}$	ft/s
0	0.00	0	0.00	0	0.00	0.00	—	1.04	—	—
3	0.00	0	0.00	78	0.24	1.53	0.00	1.22	0.00	0.00
6	0.00	0	0.00	113	0.66	2.21	0.00	1.29	0.00	0.00
9	2.99	78	0.24	141	1.16	2.77	0.09	1.29	0.15	0.88
12	5.99	112	0.66	167	1.71	3.28	0.20	1.26	0.27	1.56
15	8.99	141	1.16	193	2.27	3.78	0.31	1.22	0.37	2.14
18	11.99	167	1.71	219	2.82	4.30	0.40	1.19	0.46	2.63
21	14.99	193	2.27	247	3.32	4.86	0.47	1.15	0.52	3.02
24	17.99	219	2.81	282	3.73	5.54	0.51	1.09	0.58	3.36
27	20.99	247	3.32	360	3.98	7.07	0.47	1.00	0.60	3.48

These results provide the necessary information to construct a pipe curve on a scattergraph, as shown below.



The conditions observed within this sewer are effectively described by the Manning Equation fitted to observed flow depth and velocity data using the Stevens-Schutzbach Method with a varying roughness coefficient. Previous results using a constant roughness coefficient are shown for comparison.[2]

(d) Calculate Q_{SS} for $d = D$

The full-pipe capacity is calculated using the Continuity Equation, $Q_{SS} = Av_{SS}$, where Q_{SS} = 13.9 ft^3/s or 9.0 MGD, about 19% greater than the corresponding value determined with a constant roughness coefficient.[2]

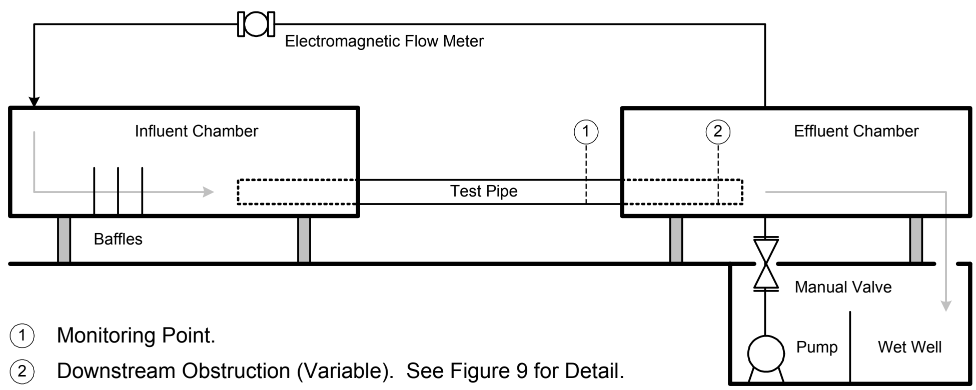
Laboratory Investigation

Laboratory investigations were previously reported to demonstrate the performance of the Design Method, the Lanfear-Coll Method, and the Stevens-Schutzbach Method under controlled conditions.[2] The results of these investigations can also be used to compare the use of a constant and varying roughness coefficient.

Equipment and Methodology

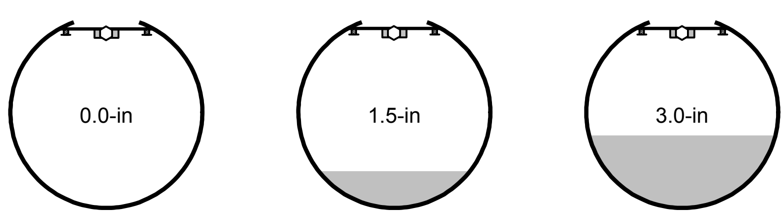
The laboratory equipment used during this investigation was designed and configured to simulate hydraulic conditions encountered in the urban sewer environment. The general arrangement of this equipment is provided in Figure 8.

FIGURE 8: Laboratory General Arrangement



A pump provides flow through a 6-in PVC force main to an influent chamber. A manual valve regulates the pump, and an electromagnetic flow meter measures the pump discharge. Flow passes through three consecutive baffles within the influent chamber, minimizing surface disturbances before entering an 8-in PVC test pipe. Uniform and non-uniform flow conditions are observed and measured at a monitoring point located within the test pipe. Flow conditions are controlled using one of three obstructions of known depth, as depicted in Figure 9, positioned a fixed distance downstream from the monitoring point. Following discharge from the test pipe to an effluent chamber, the flow is returned to a wet well for re-circulation by the pump.

FIGURE 9: Downstream Obstructions for Laboratory Investigation

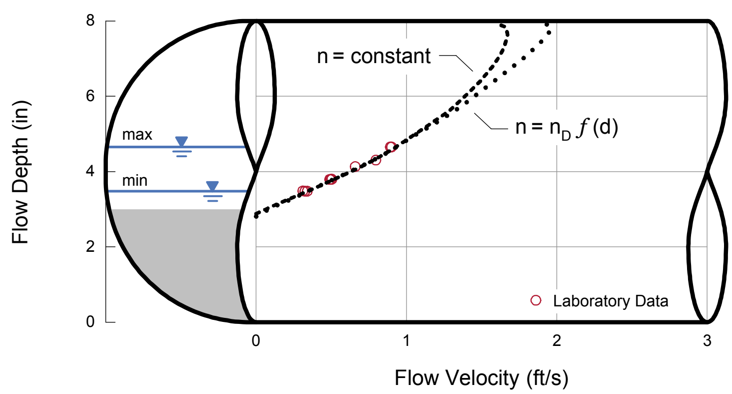


After placing an obstruction within the test pipe, the pump is activated, and flow is introduced into the system. Once the system has reached equilibrium, flow depth and quantity measurements are obtained at three consecutive one-minute intervals. Flow depth is measured in the test pipe with a stainless steel ruler, and flow quantity is measured in the force main with the electromagnetic flow meter. These measurements are then used to calculate flow velocity in the test pipe using the Continuity Equation. A total of 30 flow depth and quantity measurements were obtained at a variety of pump settings.

Results and Discussion

Flow depth and velocity data obtained using the 3.0-in downstream obstruction are plotted on a scattergraph and evaluated with respect to the Manning Equation using the Stevens-Schutzbach Method. This method is applied to the first 15 laboratory observations using both a constant and varying roughness coefficient as shown in Figure 10.

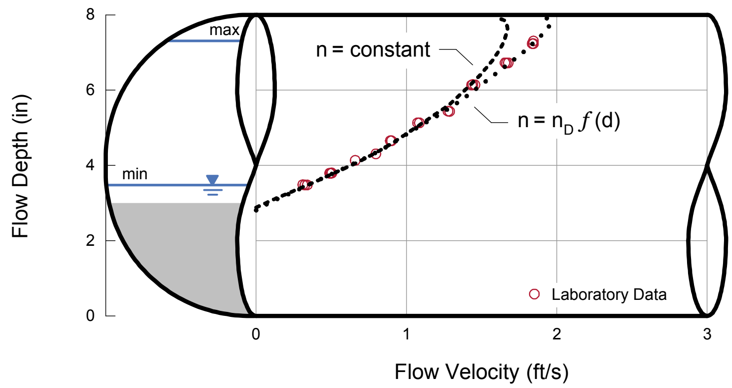
FIGURE 10: Laboratory Observations (15 of 30)



Note that these observations are effectively described using either a constant or varying roughness coefficient. The sum of the squared error (SSE) for the Stevens-Schutzbach Method using a constant or varying roughness coefficient is 0.01 $(ft/s)^2$, but which assumption provides the best estimate of actual sewer capacity? For a varying roughness coefficient, the projected full-pipe velocity is 1.94 ft/s, with an estimated sewer capacity of 0.424 MGD – about 20% greater than the corresponding values determined using a constant roughness coefficient.

To further test the two assumptions, the remaining 15 laboratory observations are added to the scattergraph and compared with the existing pipe curves from Figure 9, as shown in Figure 11.

FIGURE 11: Laboratory Observations (30 of 30)



The SSE for these observations is 0.19 $(ft/s)^2$ using a constant roughness coefficient, while the SSE is 0.01 $(ft/s)^2$ using a varying roughness coefficient. The SSE for the constant roughness coefficient is 19 times greater than the SSE for the varying roughness coefficient. These results indicate that the varying roughness coefficient provides a more accurate projection of sewer capacity than the constant roughness coefficient under these test conditions.

Conclusion

The scattergraph is a graphical tool that provides insight into sewer performance through a simple and intuitive display of flow monitor data. The Manning Equation is an important component of the scattergraph and can be applied using a variety of methods. The Design Method uses the Manning Equation to describe a relationship between flow depth and velocity using a specified roughness coefficient and pipe slope. This relationship is then compared with actual flow monitor data. The Lanfear-Coll Method and the Stevens-Schutzbach Method use curve fitting techniques to correlate the Manning Equation directly to such data. Modifications are presented to incorporate a varying roughness coefficient into these methods. The selection of a constant or varying roughness coefficient can impact sewer capacity estimates by over 20%. Laboratory results indicate that the use of a varying roughness coefficient provides a more accurate determination of sewer capacity.

Symbols and Notation

The following symbols and notation are used in this paper:

VARIABLES

- d = flow depth, in or ft
- v = flow velocity, ft/s
- Q = flow quantity, ft^3/s or MGD
- n = roughness coefficient
- R = hydraulic radius, ft
- S = slope of the energy gradient
- C = hydraulic coefficient
- D = diameter, in or ft
- A = wetted area, ft^2
- P = wetted perimeter, ft
- R^2 = coefficient of determination

SUBSCRIPTS

- $_{DM}$ = Design Method
- $_{LC}$ = Lanfear-Coll Method
- $_{SS}$ = Stevens-Schutzbach Method
- $_{dog}$ = *dead dog*
- $_{e}$ = effective
- $_{avg}$ = average
- $_{min}$ = minimum

Acknowledgement

The authors acknowledge Hal Kimbrough for his statistical guidance. Special thanks are also extended to Patrick Stevens and Paul Mitchell for their technical review.

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